Exam. Code: 0939
Sub. Code: 7045

1119
B.E. (Mechanical Engineering)
Third Semester
AS-301: MATHS-3

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.*

x-x-x

1 (a) State *nth* term test for divergence of a series. Examine whether the series $\sum_{n=1}^{\infty}\frac{n+1}{n}$ is convergent or divergent. $(5\times 2=10)$

(b) Define alternating series with a suitable example. How do you know if a series is alternating? Explain absolute and conditional converge with suitable examples.

(c) Explain row pivoting in connection with Gauss elimination method. Why do we do pivoting?

(d) Find all values of z for which $e^z=1+i$.

(d) Find the nature and type of singularity for $f(z)=\frac{e^z}{z+\sin z}$.

PART-A

2. (a) Examine the convergence or divergence of the following sequences:

(i) $a_n=\tan^{-1} n$, (ii) $a_n=\ln n-\ln(n+1)$, (iii) $a_n=n-\sqrt{n^2-n}$, (iv) $a_n=(n)^{\frac{1}{n}}$,

(b) Examine the convergence or divergence of the following series:

(i) $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$, (ii) $\sum_{n=1}^{\infty}\frac{1}{n^p}(p\ is\ a\ real\ consant)$, (iii) $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$.

3. (a): State and prove Leibnitz test for alternating series. Examine the convergence of divergence of he series: $\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt{n}+(-1)^n}$.

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(b) Find the column rank of the matrix: $A=\begin{bmatrix}1 & 3 & 2 & 2\\ 3 & 7 & 4 & 8\\ 2 & 4 & 3 & 4\\ 1 & 2 & 1 & 3\end{bmatrix}$.

4. (a) Test the consistency of the system: $x+2y+z=3,\ 2x+y+3z=5,\ 2x+4y+2z=7$.

(b) Examine whether $A=\begin{bmatrix}1 & 2 & 2\\ 0 & 2 & 1\\ -1 & 2 & 2\end{bmatrix}$ is diagonalizable or not? If yes, obtain the

matrix P such that $P^{-1}AP$ is a diagonalizable. $(3+4+3)$

(c) Examine whether A is similar to B, where $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$.

PART-B

5. (a) When a function $w=f(z)$ is said to be analytic at a point $z=z_0$? What are the necessary and sufficient conditions for a function $f(z)$ to be analytic at $z=z_0$?

(b) Examine the analyticity of the functions: (i) $xy+iy$, (ii) $z+\bar{z}$, (iii) $\sinh z$.

(c) Find Cauchy-Riemann equation in Polar form. $(3+3+4)$

6 (a) Define harmonic function. What is meant by harmonic conjugate? Prove that the function: $u(x,y)=\log\sqrt{(x^2+y^2)}$ is harmonic and find its harmonic conjugate and the function $f(z)$.

(b) State Laurent's series. Obtain the same for the function $f(z)=\dfrac{(z-1)}{(z-2)(z-3)}$ in (i) $2<|z|<3$ (ii) $|z|>3$.

7. (a) Prove that a bilinear transformation maps circles into circles or straight line. Also find a bilinear transformation which maps $0, i, -i$ into $1, -1, 0$. Also find the image of $|z|<1$ under it.

(b) Evaluate $\int_0^{\pi} \frac{1}{5+4\cos\theta}\, d\theta$ using complex integral.

x-x-x