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Exam.Code:1014
Sub. Code: 7751

1129
M.E. (Mechanical Engineering)
First Semester
MME-103: Continnum Machine

Time allowed: 3 Hours **Max. Marks: 50**

***NOTE**: Attempt <u>five</u> questions in all, selecting atleast two questions from each Section. Use usual notations and symbols for derivations. Assume suitable missing data if any.*

x-x-x

Section A

Q.1 (a) Verify in direct notation that $(\mathbf{ST})^{-1} = \mathbf{S}^{-1}\mathbf{T}^{-1}$, and (b) prove in direct notation that $(\mathbf{S}^{-1})^T = (\mathbf{S}^T)^{-1} \equiv \mathbf{S}^{-T}$.

Q.2 (a)Determine the covariant basis $\mathbf{g}_i = \mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3$ of the cylindrical polar coordinate system $(\theta^1 = r, \theta^2 = \theta, \theta^3 = z)$. (b) Also determine the Christoffel symbols Γ^i_{jk}.

Q.3 Given the motion $x_1 = 2X_2$, $x_2 = 5X_3$, $x_3 = X_1$, (a) calculate the principal stretches λ_1, λ_2, and λ_3 (common to both ($\mathbf{U}$ and $\mathbf{V}$); (b) calculate the principal directions $\mathbf{N}_1$, $\mathbf{N}_2$, and $\mathbf{N}_3$ of $\mathbf{U}$; (c) calculate the principal directions $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ of $\mathbf{V}$; (d) verify that the principal directions of $\mathbf{U}$ and $\mathbf{V}$ differ by a rotation,i.e., $\mathbf{n}_1 = \mathbf{R}\mathbf{N}_1$, $\mathbf{n}_2 = \mathbf{R}\mathbf{N}_2$, $\mathbf{n}_3 = \mathbf{R}\mathbf{N}_3$.

Q.4 In direct notation prove that $\dot{\mathbf{F}} = \mathbf{L}\mathbf{F}$.

Section B

Q.5 Starting with the spatial statement of the balance of linear momentum in integral form,

$$\frac{d}{dt}\int_{\mathcal{P}} \rho \mathbf{v}\, dv = \int_{\mathcal{P}} \rho \mathbf{b}\, dv + \int_{\partial\mathcal{P}} \mathbf{t}\, da,$$

derive the corresponding pointwise form

$$\rho\dot{\mathbf{v}} = \rho\mathbf{b} + \operatorname{div}\mathbf{T}.$$

Q.6 What is invariance under superposed rigid body motions (SRBM)? Prove that $\mathbf{t}^+ = \mathbf{Q}\mathbf{t}$.

Q.7 Prove in direct notation that

$$\frac{\partial\psi}{\partial\mathbf{F}} \cdot \dot{\mathbf{F}} = \frac{\partial\psi}{\partial\mathbf{F}}\mathbf{F}^{\mathrm{T}} \cdot \mathbf{L}.$$

Q.8 Demonstrate that the Newtonian (fluid) constitutive equation can be combined with conservation of linear momentumm to obtain the Navier-Stokes equations.

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