

2056
B.E. (Mechanical Engineering)
Fourth Semester
MEC-406: Numerical Analysis

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Part. Use of a simple calculator is allowed. All questions carry 10 marks.

x-x-x

1. (a) State intermediate value property of a continuous function. Explain the similarities and differences between Bisection and Regula-falsi method.
- (b) Define well-conditioned and ill-conditioned systems. Also, define condition number of a matrix. $(5 \times 2 = 10)$
- (c) Define the different operators: Δ , ∇ and E . Establish the relation between them.
- (d) Explain Richardson extrapolation order improvement method.
- (e) Derive Euler's method for IVP geometrically. What is Euler improvement method?

PART-A

2. (a) Discuss the advantages and disadvantages of using **MATLAB** for solving engineering problems. Provide examples of how they can be applied in numerical methods. $(4 + 3 + 3)$
- (b) Define significant figures and explain their importance in numerical computations. Perform the following operations using correct significant figures: (i) 3.76×1.243 , (ii) $5.24/1.2345$.
- (c) Prove that Newton's method is quadratically convergent.
3. (a) Apply Regula-falsi and Secant method to solve $3x - \cos x - 1 = 0$ correct to three decimal places.
- (b) Discuss the potential problems in the Gaussian Elimination Method when the pivot element (diagonal element) is very small or zero. Explain how this can affect the stability of the solution. Solve the following system by partial pivoting: $2x + 17y + 4z = 35$; $x + 3y + 10z = 24$; $28x + 4y - z = 32$.
4. (a) Define and discuss the special types of matrices. Solve the system by Gauss-Seidel iteration method:

$$x + 2y + 3z = 10; 2x + 3y + 3z = 13; 4x + y + z = 7.$$

(2)

- (b) Using 1964 as the origin obtain a straight line trend equation by the method of least squares:

Year	1960	1962	1963	1964	1965	1966	1969
Value	140	144	160	152	168	176	180

Find the trend value of the missing year 1961.

PART-B

5. (a) Compare different interpolation formulas. Given $f(0) = -18, f(1) = 0, f(3) = 0, f(5) = -248, f(6) = 0, f(9) = 13104$, find $f(x)$ and $f(2)$.
- (b) Explain the applications of inverse interpolation. Find the value of x for $y = f(x) = 3000$ by inverse interpolation from the data:
- | | | | |
|-----------|------|------|------|
| x: | 10 | 15 | 20 |
| y = f(x): | 1754 | 2648 | 3564 |
6. (a) Compare basic numerical integration rules namely Trapezoidal rule, Simpson's one-third and three-eighth rule. Compute $\int_1^2 \frac{2x}{1+x^4} dx$ using them.
- (b) Using finite difference methods, calculate the first and second-order partial derivatives of the function $f(x, y) = e^{x^2+y^2}$ at the point (1,1). Use a step size of $h = 0.01$. Use Richardson extrapolation to enhance the accuracy.
7. (a) Use the Runge-Kutta method to solve the ODE $\frac{dy}{dx} = x^2 - y^2$, with initial condition $y(0) = 0$ over the interval $[0, 2]$.
- (b) Consider the boundary value problem $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ with boundary conditions $y(0) = 1$ and $y(1) = 0$. Use the finite difference method with 4 grid points to approximate the solution.

x-x-x