Exam. Code: 0922
Sub. Code: 33526

2056

B.E. (Information Technology) Fourth Semester
ASM-401: Discrete Structures

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt five questions in all, including Question No. 1(Section-A) which is compulsory and selecting two questions each from Section B-C.***

x-x-x

Section - A

1. Answer the following:

 a) In how many ways can 15 candy bars be distributed among five children so that the youngest gets only one or two of them?

 b) Define group and ring.

 c) Define total order set and Lexicographic order with examples.

 d) Determine whether $(p \wedge (p \to q)) \wedge \neg q$ is a tautology or contradiction.

 e) If $f(x) = 2x + 3$ and $g(x) = x^2 + 1$, find $f \circ g$ and $g \circ f$. Also, find $f^{-1}(x)$.

$(5 \times 2 = 10)$

Section - B

2. a) Define a relation $\sim$ on $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ by $(x_1, x_2) \sim (y_1, y_2)$ if $x_1 + x_2 = y_1 + y_2$. Prove that it is an equivalence relation and describe the equivalence class of (1,2).

 b) In a group of 6 people, each pair of individuals are either acquaintances or strangers. Prove that there exist at least 3 people who are either all mutual acquaintances or all mutual strangers.

 c) Let S be a set of 6 positive integers, each not exceeding 15. Show that the sums of elements of all possible non-empty subsets of S cannot be all distinct.

$(03 + 03 + 04)$

3. a) Consider the set $A = \mathbb{R}^2 - \{(0,0)\}$. Define a relation $\sim$ on A by $(x_1, x_2) \sim (y_1, y_2)$ if there exists $t > 0$ such that $x_1 = ty_1$ and $x_2 = ty_2$. Prove that $\sim$ is an equivalence relation. Also, find the equivalence class of (2, 3).

 b) If $(P_1, \preceq)$ and $(P_2, \preceq)$ are pastial ordered sets (POSETs) then show that $(P, \preceq)$ is also a POSET, where $P = P_1 \times P_2$ and the partial order $\preceq$ of P is the product partial order.

$(05 + 05)$

P.T.O.

4. a) Check the validity of the argument:

If it is cloudy or the alarm fails, then Ravi will miss his morning class. Whenever the temperature is above 30°C, it is not cloudy. Today the temperature is 32°C and the alarm worked properly. Therefore Ravi will attend his morning class.

b) Check the validity of the argument:

$$\begin{array}{l} \forall x, [p(x) \rightarrow q(x)] \\ \exists x,\ p(x) \\ \forall x, [q(x) \rightarrow r(x)] \\ \forall x, [r(x) \rightarrow \neg\ s(x)] \\ \hline \therefore\ \exists\ x,\ \neg\ s(x) \end{array}$$

(05 + 05)

Section - C

5. a) Define the following with suitable examples:

i. Bipartiate Graph

ii. Chromatic number of graph

iii. Euler circuit and Hamiltonian circuit

iv. Isomorphic graphs

b) What is the chromatic polynomial for $K_{2,3}$? What is the chromatic number of $K_{2,3}$?

(05 + 05)

6. a) In how many ways can 20 identical balls be distributed among 5 distinct persons such that each person receives at least 2 balls but not more than 7 balls?

b) Solve the following recurrence relation using method of generating functions:

$$a_{n+2} - 5a_{n+1} + 6a_n = 2,\ n \geq 0, \quad a_0 = 3, \quad a_1 = 7.$$

(05 + 05)

7. a) If H is a subgroup of the finite group G, then for all $a, b \in G$,

(a) $|aH| = |H|$

(b) either $aH = bH$ or $aH \cap bH = \emptyset$.

b) Prove that a finite integral domain is a field. Also, show that every field is an integral domain.

(05 + 05)

x-x-x