**Exam.Code:1015**
**Sub. Code: 35216**

**2056**
**M.E. (Mechanical Engineering)**
**Second Semester**
**MME-201: Continuum Mechanics**

**Time allowed: 3 Hours** **Max. Marks: 50**

***NOTE: Attempt five questions in all, selecting atleast two questions from each Part. Use usual notations and symbols for derivations. Assume suitably missing data if any. All questions carry equal marks***

*x-x-x*

## Part A

**Question 1** Two tensors **A** and **B** are given in the form of their matrix representations

$$[A] = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad [B] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Verify that these matrices are orthogonal. Show that $[\boldsymbol{A}]$ describes a rotation and $[\boldsymbol{B}]$ a reflection. Give a geometrical interpretation.

**Question 2** Apply the operator $\nabla$ to products of smooth vector fields $u$ and $v$ to establish the following identity

$$\boldsymbol{\nabla} \cdot (\boldsymbol{u} \times \boldsymbol{v}) = \boldsymbol{v} \cdot \boldsymbol{\nabla} \times \boldsymbol{u} - \boldsymbol{u} \cdot \boldsymbol{\nabla} \times \boldsymbol{v}$$

**Question 3** In a deformation of a three-dimensional problem, the displacement components of $\boldsymbol{u}$ are found to be

$$u_1 = x_1 - \frac{1}{4}x_2, \qquad u_2 = x_1 + 2x_2, \qquad u_3 = -3x_3$$

Compute the matrix representation of $\boldsymbol{F}^{-1}$ and $\boldsymbol{F}$ and deduce that the deformation is *isochoric*.

**Question 4** A body undergoes uniform extension in all three directions according to

$$x_1 = \lambda_1 X_1, \qquad x_2 = \lambda_2 X_2, \qquad x_3 = \lambda_3 X_3.$$

Find the matrix representations of the tensors $\boldsymbol{F}$, $\boldsymbol{R}$, $\boldsymbol{U}$, $\boldsymbol{E}$, and $\boldsymbol{e}$.

*P.T.O.*

## Part B

**Question 5** At a certain point of a body the component of the *Cauchy* stress tensor are given by

$$[\sigma] = \begin{bmatrix} 5x_2x_3 & 3x_2^2 & 0 \\ 3x_2^2 & 0 & -x_1 \\ 0 & -x_1 & 0 \end{bmatrix} \frac{kN}{cm^2}.$$

Find the components of the *Cauchy* traction vector $\boldsymbol{t}$ at the point on the plane whose normal has direction ratious $3 : 1 : -2$. Find the normal and shear components of $\boldsymbol{t}$ on that plane.

**Question 6** The motion of a continuum body is given in the form

$$\boldsymbol{x} = (1 + \alpha(t)t)\boldsymbol{X},$$

with the scalar function $\alpha(t)$ and the set $\{\boldsymbol{e}_a\}, a = 1, 2, 3$ of orthogonal unit vectors. Find the expression for the spatial mass density $\rho$ in terms of $\rho_o$ so that the continuity equation is satisfied.

**Question 7** Starting from the global form of the second law of thermodynamics, show that the local spatial form of the second law is

$$\frac{\partial \eta}{\partial t} \geq -\boldsymbol{\nabla} \cdot (\boldsymbol{h} + \eta \boldsymbol{v}) + r$$

**Question 8** In the context of *Objectivity*, What is a *Euclidean transformation*? When is a spatial vector field said to be *objective*? Show that the spatial velocity field is *not* objective. Similarly determine the transformation for the spatial acceleration field and identify the quantities that correspind to the *Euler acceleration*, *centrifugal acceleration*, and *Coriolis acceleration*. Is the acceleration field objective?

x-x-x