Exam. Code: 0927
Sub. Code: 33641

2076

B.E. (Electronics and Communication Engineering)
Third Semester
MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.***

x-x-x

1. (a) Define spanning set of vector space. Find the span of vectors $(1, 0, 1)$ and $(2, 1, 3)$ in R^3.

(b) Define linear transformation and verify whether $T(x, y)=(x+y, x-y)$ is linear.

(c) Define eigenvalue problem of a matrix. Find the trace and determinant of a matrix with eigenvalues $1, 4$ and 7.

(d) Distinguish between differentiability and analyticity.

(e) Define conformal mapping. Show that $w = z + c$ is conformal.

SECTION-A

2. (a) Define homogeneous system of linear equations. Show that a homogeneous system always has a trivial solution. Find non-trivial solution if possible:

$x + 2y + z = 0;\ 2x + 4y + 2z = 0.$ $(4 + 3 + 3)$

(b) Explain relation between rank of coefficient matrix and consistency of system. Also, define row echelon form and reduced row echelon form with examples.

(c) Prove intersection of two subspaces is a subspace. Give a counterexample for union.

3. (a) Prove or disprove: $V = \{(x, y) \in R^2 : x + y = 1\}$ is a vector space over R.

(b) Define a linear combination of vectors. Illustrate the concept with suitable examples. Explain its significance in vector spaces.

(c) Diagonalize the matrix: $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. $(2 + 3 + 5)$

P.T.O.

4. (a) Consider the matrix mapping: A: $R^4 \rightarrow R^3$, where $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & 3 \end{bmatrix}$. Find basis and dimension of (i) the image of A, (ii) the kernel of A.

(b) The vectors $u = (1, 2, 0), v = (1, 3, 2), w = (0, 1, 3)$ form a basis S of R^3.

Find (i) the change of basis matrix P from the usual basis $E = \{e_1, e_2, e_3\}$,

(ii) The change of basis matrix Q from S back to E.

SECTION-B

5. (a) Separate $\log(x + iy)$ into real and imaginary parts.

(b) Find all the values of z for which $\sin z = 200$. $(3 + 4 + 3)$

(c) Show that: $f(z) = |z|^2$ is continuous everywhere but not differentiable anywhere except possibly at $z = 0$.

6. (a) Explain the relationship between analyticity and harmonicity. Show that the real and imaginary part of an analytic function are harmonic.

(b) Explain the relationship between zeros, poles and residues of a function of complex variable. $(4 + 3 + 3)$

(c) Find residues of $f(z) = \frac{e^{z^2}}{(z-i)^3}$ at its pole.

7. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{\cos\theta + 2}$ using complex integration.

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into points $w = i, 0, -i$. Hence, find the image of $|z| < 1$.

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