Exam.Code:0915
Sub. Code: 33423

2076

B.E. (Computer Science and Engineering)
Third Semester
CS-303: Discrete Structures

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section.*

x-x-x

1. Briefly explain the following with example:

(a) Antisymmetric relation

(b) Planar graph

(c) Ordinary generating functions

(d) Groups

(e) Universal quantifier

(5x2=10)

Section-A

(2) (a) Each of the following defines a relation on the positive integers $\mathbb{N}$
(i) $x > y$ is the square of an integer
(ii) $x + 4y = 10$
Determine which relations are reflexive, symmetric, antisymmetric, and transitive. **(6)**

(b) If A and B be any sets, then show that A is the disjoint union of $A \setminus B$ and $A \cap B$. **(4)**

(3) (a)

(b) Let $f : [0, \infty) \to \mathbb{R}$ be a function defined by:

$$f(x) = x^2 + 2x + 1$$

Prove that f is one-one but not onto **(6)**

(c) Prove that there is no largest integer that is a perfect square using a suitable method of proof. **(4)**

P.T.O.

(4) (a) Let $D_{48} = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$, where all the elements are divisors of 48. Let the relation | (divides) be a partial ordering on D_{48}.

(i) Draw the Hasse diagram for the poset defined by the divisors of 48.

(ii) Find the least upper bound (lub) of the sets $\{2, 3\}$ and $\{12, 24\}$.

(iii) Determine if the element 16 is an upper bound for the set $\{6, 12\}$. **(6)**

1. Check the validity of the following argument using rules of inference of propositional logic without using truth tables:

   Argument: If the alarm rings, then Sam wakes up. Whenever Sam wakes up, he feels refreshed. Today, Sam feels refreshed. Hence, the alarm must have rung. **(4)**

Section-B

(5) (a) Solve the recurrence relation:

$$2b_{n+1} - 5b_n + 3b_{n-1} = 1 + n^2 \quad \text{for } n \geq 2.$$

(6)

(b) Solve the recurrence relation:

$$c_n - 3c_{n-1} = 2^n \quad \text{for } n \geq 2$$

using the method of generating functions. **(4)**

(6) (a) Consider the graph G in following figure.

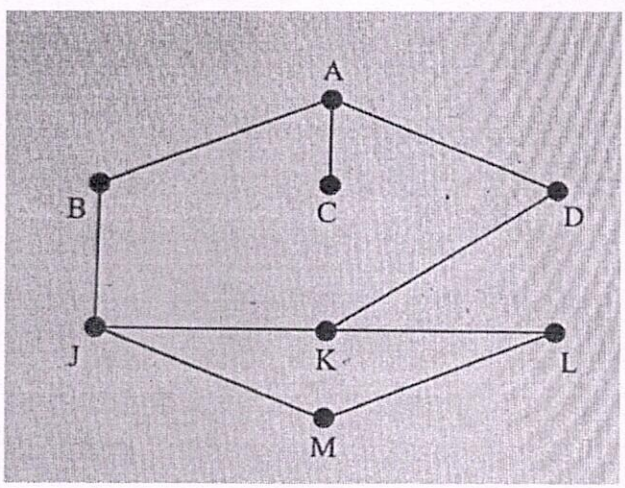

Figure 1:

(i) Find the adjacency structure of G.
(ii) Find the order of the vertices of G is processed using a Breadth First Search algorithm beginning at vertex A. **(6)**

(b) Define a complete bipartite graph $K_{m,n}$. Provide an example and explain how to determine the number of edges in $K_{m,n}$. **(4)**

(7) Consider the set $\mathbb{Q}$ of rational numbers, and let $\circ$ be the operation on $\mathbb{Q}$ defined by:

$$a \circ b = a + b + ab - 1$$

Find the following:

(i) $1 \circ 2$

(ii) $\frac{1}{3} \circ 0$

(iii) Is $(\mathbb{Q}, \circ)$ a semigroup? Is it commutative?

(iv) Find the identity element for $\circ$.

(v) Do any of the elements in $\mathbb{Q}$ have an inverse under operation $\circ$? If so, what is it?

(10)

x-x-x