Exam.Code:1014
Sub. Code: 35201

2076
M.E. (Mechanical Engineering)
First Semester
MME-101: Advanced Engineering Mathematics

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, selecting atleast two questions from each Section. Use of Simple calculator is allowed. All questions carry equal marks.

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SECTION-A

1. (a) Using the power series method, obtain the solution of :

$$(1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-y=0 \text{ about } x=0.$$

(b) Using Frobenius method, solve $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+(x^2-4)y=0$ and obtain the indicial equation.

2. (a) Derive Rodergues formula for Legendre polynomials and prove that $P_n(1)=1$.

(b) Obtain Frobenius solution of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$.

3. (a) Define Bessel's functions of first and second kind. Prove that

$$\int J_3(x)\,dx + J_2(x) + \frac{2}{x}J_1(x) = 0.$$

(b) Show that the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(k^2x^2-n^2)y=0$.

can be transformed into Bessel's differential equation by using the substitution $z = kx$. Hence, find its solution.

4. (a) Explain Sturm-Liouville eigenvalue problems and develop eigen-function expansions.

(b) Find the eigen-functions of Sturm-Liouville equation: $\frac{d^2y}{dx^2}+\lambda y=0$,

$$y(\pi)=y(-\pi), \frac{dy}{dx}(\pi)=\frac{dy}{dx}(-\pi).$$

SECTION-B

5. (a) Explain the extension of the Runge–Kutta fourth-order method for solving a system of first-order differential equations. Hence, solve $\frac{dx}{dt}=x+y, \frac{dy}{dt}=x-y$ with $x(0)=1, y(0)=0$, taking $h=0.1$.

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(b) Explain the procedure for converting a higher-order differential equation into a system of first-order equations. Develop a numerical algorithm for solving the resulting system.

6. (a) Derive the finite difference scheme for a second-order BVP and discuss truncation error.

(b) Solve two-dimensional Laplace equation in the square region $0 \leq x \leq 2$, $0 \leq y \leq 2$ using five-point finite difference formula with mesh size $h = 1$. The boundary values are given as $u(x,0) = 0, u(0,y) = 0, u(x,2) = 100, u(2,y) = 100$. Find the values of u at the grid points.

7. State advantages of the Crank–Nicolson method over the explicit finite difference method. Solve $u_t = 2\,u_{xx}$ for $0 \leq x \leq 4$ with $u(0,t) = 0$, $u(4,t) = 0$ and $u(x,0) = 4\sin\frac{\pi x}{4}$. Take $h = 1, k = 0.25$. Determine the temperature distribution after one time step using the implicit finite difference method.

8. (a) Compare the finite difference schemes used for parabolic and hyperbolic equations. $(4 + 6)$

(b) Solve $u_{tt} = 9u_{xx}$ for $0 \leq x \leq 3$, subject to $u(0,t) = 0, u(3,t) = 0$, with initial conditions $u(x,0) = 3x - x^2$, $u_t(x,0) = 0$. Take $h = 1, k = \frac{1}{3}$. Find the displacement at the first time level.