**Exam.Code:0905**
**Sub. Code: 33241**

**2076**
**B.E., First Semester**
**ASM-101: Calculus**
**(Common to all streams)**

**Time allowed: 3 Hours** **Max. Marks: 50**

***NOTE:*** ***Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry 10 marks.***

*x-x-x*

1. (a) Explain the difference between sequence and series. State Leibnitz for alternating series.
   (b) Write formula for surface of revolution about x-axis. Illustrate it with suitable example.
   (c) Write Taylor's theorem for two variables. Explain its significance.
   (d) Define double integral as a limit of sum.
   (e) What are solenoidal and irrotational vector fields?

**SECTION-A**

2. (a) Which of the following sequences convergence and which divergence? Find the limit of each convergent sequence:

   (i) $a_n = \left(\frac{n+1}{n-1}\right)^n$ $n \geq 2$, (ii) $a_n = \left(1 - \frac{1}{n}\right)^n$, (ii) $a_n = \sqrt[n]{2^n\, n}$, (iv) $a_n = \frac{8n!}{4^n.2^n}$.

   (b) State and prove Cauchy's integral test. Discuss the convergence of the series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$, where p is a real constant.

3. (a) Discuss the convergence or divergence of he following series:

   (i) $\sum_{n=1}^{\infty} \frac{n\,(n+1)}{(n+2)\,(n+3)}$, (ii) $\sum_{n=1}^{\infty} \frac{5^n}{4^n+3}$, (iii) $\sum_{n=1}^{\infty} \frac{1+cosn}{n^2}$.

   (b) The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

4. (a) Establish the fact that if $f(x, y, z) = 0$, then $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$.

   (b) Find the absolute maximum and minimum values of the function $f(x, y)$ on the triangular region in the first quadrant bounded by the lines $x = 0, y = 0$ and $y = 9 - x$, where $f(x, y) = 2 + 2x + 4y - x^2 - y^2$.

**SECTION-B**

5. (a) Let $y = f(x)$ be a given curve, where f is twice differentiable. Derive the formula for curvature. Also, prove that the curvature is zero at a point of inflection.

   (b) If $\vec{a}$ is a constant vector and $\vec{r} = (x)\hat{\imath} + (y)\,\hat{\jmath} + (z)\hat{k}$, show that

$$\text{curl}(\vec{a} \times \vec{r}) = 2\,\vec{a}.$$

6. (a) A fluid's velocity field is $\vec{F} = x\,\hat{\imath} + z\,\hat{\jmath} + y\,\hat{k}$. Find the flow along the helix $\vec{r} = (cost)\hat{\imath} + (\,sint)\,\hat{\jmath} + (t)\hat{k}, 0 \leq t \leq \frac{\pi}{2}$.

(b) Show that $\vec{F} = (e^x cosy + y\,z)\hat{\imath} + (xz - e^x\,siny)\,\hat{\jmath} + (xy + z)\,\hat{k}$, is conservative over its natural domain and find a potential for it.

7. (a) State Green's theorem. Calculate the outward flux of the vector field $\vec{F}(x, y) = 2e^{x\,y}\,\hat{\imath} + y^3\,\hat{\jmath}$ across the boundary of the square $-1 \leq x \leq 1, -1 \leq y \leq 1$.

(b) Use Stoke's theorem to evaluate $\oint \vec{F} \cdot d\vec{r}$, if $\vec{F} = (xz)\hat{\imath} + (xy)\,\hat{\jmath} + (3xz)\,\hat{k}$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, traversed counterclockwise as viewed from above.

x-x-x