Exam.Code:0918
Sub. Code: 33443

2055

B.E. (Computer Science and Engineering)
Sixth Semester
CS-602: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. Use of a simple calculator and statistical tables is allowed. All questions carry10 marks.

x-x-x

1. (a) Prove that a homogeneous system with more unknowns than equations has infinitely many solutions.

(b) Prove that the intersection of two sub-spaces is a subspace, but the union need not be.

(c) Show that the eigenvalues of A and A^T are the same. Do they share the same eigenvectors?

(d) Differentiate between discrete and continuous random variable. What is the expectation of a discrete random variable?

(e) List out four properties of the Poisson distribution.

SECTION-A

2. (a) Define a vector space. Prove that the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space.

(b) Let U and V be two subspace of R^4 given by $U = \{(a, b, c, d): b - 2c + d = 0\}$ and $V = \{(a, b, c, d): a = d, b = 2\}$. Find the basis and dimension of V and $U \cap V$.

3. (a) Solve the homogeneous system $AX = 0$, where $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 1 & 4 \\ 3 & 2 & -6 & 1 \end{bmatrix}$. Also find the rank and nullity of A.

(b) Define eigenvalue problem of matrices. Find the eigenvalues and and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. Is it diagonalizable?

4. (a) Let $T: R^4 \to R^3$ be the linear transformation defined by $T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$. Find the basis and dimension of (i) Range of T, Null space of T. Also, verify rank-nullity theorem.

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(b) Let T be a linear operator on R^2 defined by $T(x,y) = (4x - 2y, 2x + y)$. Find (i) the matrix of T relative to the basis $B = \{(1,1), (-1,0)\}$, (ii) Also verify $[T:B][v:B] = [T(v):B]$ for every $v \in R^2$.

SECTION-B

5. (a) A candidate has to reach the examination centre in time. Probability of his going by bus or scooter or by other means of transport is $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability of his getting late are $\frac{1}{4}$ and $\frac{1}{3}$, respectively if he travels by bus or scooter. But, he reaches in time if he uses any other mode of transport. If he reached late at the centre, find the probability that he travelled by bus.

(b) A random experiment consists of throwing two unbiased dice. Let random variable X denotes the sum of faces. Find cumulative distribution function of X.

6. (a) Let X be a random variable having probability distribution function: $f(x) = 6x(1 - x), 0 < x < 1$and zero elsewhere. Then, find (i) Expectation, (ii) Variance of X, (iii) $P(\mu - 2\sigma < X < \mu + 2\sigma)$.

(b) In a normal distribution, 31% items are below 45 and 8% are above. Find the mean and standard deviation of the distribution.

7. (a) Prove that the random variables X and Y with joint probability density function: $f(x,y) = 12xy(1 - y), 0 < x < 1, 0 < y < 1$ and zero elsewhere, are independent.

(b) A straight line of length 4 units is given. Two points are taken at random on this line. Find the probability that the distance between them is greater than 3units.

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