Exam. Code: 0940
Sub. Code: 33860

2055
B.E. (Mechanical Engineering)
Fourth Semester
MEC-406: Numerical Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Part. Use of a simple calculator is allowed. All questions carry 10 marks.***

x-x-x

1. (a) Define truncation error and round-off error with an example. Why does error propagation occur in numerical computations? Give an example.

   (b) Define well-conditioned and ill-conditioned linear algebraic systems with suitable examples. How to check mathematically whether the given system is ill-conditioned or well-conditioned?

   (c) Define curve fitting and distinguish it from interpolation. What is the difference between linear and non-linear regression?

   (d) A student computes $\int_0^2 x^3\, dx$ using Simpson's rule with $n = 4$ and gets an error of 0.5. Is this reasonable? Justify.

   (e) For the IVP: $\frac{dy}{dt} = -2y$, calculate $y(0.2)$ using Euler's method with $h = 0.5$ and $y(0) = 1$.

PART-A

2. (a) Given that $x = 20.00 \pm 0.05, y = 0.0356 \pm 0.0002, z = 15500 \pm 100$. Find the maximum value of the absolute error in (i) $x + y + z$, (ii) z^3.

   (b) Compute $log_e\,(1.02)$ truncating after the third term. Find the error.

   (c) Obtain $\sqrt{12}$ to three decimal places by using fixed point iteration method.

3. (a) Derive modified Newton's method for a root of multiplicity m. Find the multiple root of the equation: $x^3 - 5x^2 + 8x - 4 = 0$ using it.

   (b) Define norm of a matrix. List the different types of a matrix. What is a condition number of a matrix? Explain how the condition number is useful in determining whether a matrix is ill-conditioned or well-conditioned? Check the linear system for ill-conditioned $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{5}{6} \end{bmatrix}$.

4. (a) Explain Gauss elimination method with partial pivoting to solve a system of linear algebraic equation and apply it to solve the following linear systems:

$$2x + y - z = -1;\ x - 2y + 3z = 9;\ 3x - y + 5z = 14.$$

(b) Find the values of a, b, c so that $y = a + bx + c\,x^2$ is the best fit to the data:

$$\begin{bmatrix} x & 0 & 1 & 2 & 3 & 4 \\ y & 1 & 0 & 3 & 10 & 21 \end{bmatrix}.$$

PART-B

5. (a) What are the assumptions made in interpolation? The area A of a circle of diameter d is given for the following values:

$\begin{bmatrix} d: & 80 & 85 & 90 & 95 & 100 \\ A: & 5026 & 5674 & 6362 & 7088 & 7854 \end{bmatrix}$. Find approximate values for the areas of circles of diameters 82 and 91, respectively.

(b) Using the following table, find f(x) as a polynomial in powers of $(x - 6)$:

$$\begin{bmatrix} x: & -1 & 0 & 2 & 3 & 7 & 10 \\ f(x): & -11 & 1 & 1 & 1 & 141 & 561 \end{bmatrix}.$$

6. (a) Explain the principle behind the Romberg's method of integration. Apply it to evaluate $\int_0^{\frac{\pi}{2}} sinx\, dx$.

(b) Find the value of cos1.74 using the values given in the table below:

$$\begin{bmatrix} x: & 1.70 & 1.74 & 1.78 & 1.82 & 1.86 \\ sinx: & 0.9916 & 0.9857 & 0.9781 & 0.9691 & 0.9584 \end{bmatrix}.$$

7. (a) Using Runge-Kutta method of 4th order, find the approximate values of x and y at t = 0.2 for the system: $\frac{dx}{dt} = 2x + y, \frac{dy}{dt} = x - 3y, t = 0, x = 0, y = 0.5.$

(b) Using Milne's method, compute y(4.4) given that: $5\,x\,\frac{dy}{dx} + y^2 = 2, y(4) = 1,$

$y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143.$

x-x-x