

2055

B.E. (Electronics and Communication Engineering)

Fourth Semester

EC-407: Probability and Random Processes

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.

x-x-x

- Q.1a) Define central limit theorem. (5×2)
- b) When is a stochastic process said to be ergodic.
- c) Suppose a coin is flipped 3 times. What is the probability of getting two tails and one head?
- d) Write down the properties of power spectral density.
- e) Why flicker noise is called low frequency noise?

Section - A

- Q.2a) Let X be a continuous random variable with pdf (3)

$$f(x) = \begin{cases} ax & ; 0 \leq x \leq 1 \\ a & ; 1 \leq x \leq 2 \\ -ax + 3a & ; 2 \leq x \leq 3 \\ 0 & ; \text{otherwise} \end{cases}$$

- (i) Determine the constant a; (ii) $P(X \leq 2.5)$
- b) Bayes theorem is a combination of addition and multiplication theorem. Justify with the suitable mathematics. (4)
- c) Discuss briefly: Properties of Hilbert transform, Bandwidth of HPF. (3)
- Q.3a) If $Z = aX + bY$, where a and b are constants. Prove: $EZ = aEX + bEY$ (E represents Expected value). (5)
- b) The joint probability distribution of X and Y is given by, (5)

$$f(x, y) = \frac{1}{27} (2x + y); x: 0, 1, 2 \text{ and } y: 0, 1, 2$$

(i) Find the marginal distributions of X and Y. (ii) Are X and Y independent random variables.

- Q.4a) X(t) and Y(t) are the input and output of LTI system respectively. If X(t) is wide sense stationary process, comment on the stationarity of the autocorrelation of Y(t). (5)
- b) The number of calls received in a telephone exchange follows a Poisson distribution with an average of 10 calls per minute. What is the probability that in one-minute duration, (5)
- (i) no call is received (ii) exactly 5 calls are received.

Section - B

- Q.5a) Apply the Shannon Fano coding to find the efficiency of the following message ensemble: (6)
- [X] = [x1, x2, x3, x4, x5, x6, x7]
- [P] = [0.4, 0.12, 0.12, 0.08, 0.08, 0.04, 0.16]
- b) What are discrete memoryless channels? Find the capacity of the Binary Symmetric Channel when probability p is given as 0.6. (4)

(2)

- Q.6a) "For a fixed signal power, and in the presence of white Gaussian Noise, the channel capacity approaches an upper limit with Bandwidth increased to infinity." Explain the related Trade-off. (5)
- b) A signal is band limited to 8kHz. The signal is quantized in 6 levels of a system with respective probabilities as 0.2, 0.1, 0.1, 0.05, 0.05 and 0.05. Calculate the entropy and the rate of information. (5)
- Q.7a) What is entropy? Verify that: $H(XY) = H(Y/X) + H(X)$ where H represent the entropy and X, Y are transmitter and receiver respectively. (5)
- b) In a cascade amplifier, show that the contribution to overall noise-figure is primarily by the first stage and contribution by succeeding stages becomes smaller and smaller. (5)

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