Exam. Code: 0916
Sub. Code: 33418

2055

B.E. (Computer Science and Engineering)
Fourth Semester
CS-402: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. Use of simple calculator and statistical table is allowed. All questions carry 10 marks.

x-x-x

1. (a) What is the span of a set of vectors? Is $(0,0,0)$ in any span? Does $\{(1,2),(2,4)\}$ span R^2?

(b) Define the kernel of a linear mapping. Does the kernel of a linear map is always a subspace of the co-domain?

(c) If A is diagonalizable, then prove that A^k is diagonalizable for all $k \in N$.

(d) Compute $E[X^3]$ for $X \sim$ Uniform $(0,1)$.

(e) State Chebyshev's inequality. Explain why this inequality is useful when the distribution of a random variable is unknown?

SECTION-A

2. (a) Find the values of μ for which the rank of the following matrix is 3:

$$A = \begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix} \qquad (3+4+3)$$

(b) Let R be the field of real numbers. Which of the following are sub-spaces of $R^3(R)$? (i) (x,y,z): x,y,z are rational numbers, (ii) $\{(x,x,x): x \in R\}$.

(c) Prove that any subset of a linearly independent set is linearly independent. Also, provide an example to illustrate this result.

3. (a) State Cayley-Hamilton theorem. Hence, find A^{78} for the matrix $A = \begin{bmatrix} 2 & -1 \\ 2 & 5 \end{bmatrix}$.

(b) Examine whether the matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ is diagonalizable or not? If yes, then diagonalize it.

4. (a) Prove that the linear transformation $T: R^2 \to R^2$ is a vector space isomorphism defined by $T(x,y) = \{x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta\}$.

(b) Find the matrix representation of the linear transformation $T: R^2 \to R^3$ defined by $T(x,y) = (3x - 2y, 0, x + 4y)$ with respect to ordered basis

$B_1 = \{(1,1),(0,2)\}$ and $B_2 = \{(1,1,0),(1,0,1),(0,1,1)\}$ for R^2 and R^3 respectively.

P.T.O.

SECTION-B

5. (a) State and prove Bayes theorem. Discuss the assumptions under which it holds.

(b) Derive the formula for the mean and variance of a linear combination of n random variables. Let X and Y are independent random variables with $E(X) = 4, Var(X) = 1, Var(Y) = 3$. Let $W = 5X + 2Y$. Compute the expected value $E(W)$ and variance.

6. (a) Explain how the normal distribution can be used to approximate the binomial distribution? What is a continuity correction and why is it necessary? Provide a worked example.

(b) Define the moment generating function of a random variable. Derive the moment generating function of a Poisson random variable. Use it to find the mean and variance.

7. (a) The joint probability distribution function of the continuous random variables X and Y is given by $f(x, y) = 2\,x\,e^{-y}, 0 < x < 1, y > 0$ and zero elsewhere. Find the distribution of $X + Y$.

(b) Describe the conditions under which the central limit theorem can be applied. Why is sample size important? A call center receives an average of 15 calls per hour with a standard deviation of 5. If the average number of calls is recorded over 36 hours, what is the probability that the mean number of calls per hour is between 14 and 16?

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