Exam. Code: 0922
Sub. Code: 33526

2055

B.E. (Information Technology) Fourth Semester
ASM-401: Discrete Structures

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Part.***

x-x-x

Question I (a) Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and consider the following equivalence relation on $A : \forall a, b \in A$, a is related to b if an only if 5 divides $a^2 - b^2$. What is the partition of A induced by this equivalence relation on A.

(b) Let R be a non-empty relation on a collection of sets defined by ARB if and only if $A \cap B = \phi$. Then which of the following is correct and why?
(i) R is reflexive and transitive
(ii) R is an equivalence relation
(iii) R is symmetric and not transitive
(iv) R is not reflexive and not symmetric

(c) Prove or disprove: k is odd integer is a necessary and sufficient condition for k^3 to be an odd integer.

(d) What do you mean by a derangement. How many derangements of 1, 2, 3, 4 are possible?

(e) What is a bipartite graph? What is the condition on a bipartite graph to be planar?
$(2 \times 5 = 10)$

Part A

Question II (a) Which of the following functions are one-one? Explain. Also determine the range of these functions.
(i) $f : \mathbb{Z} \to \mathbb{Z},\ f(x) = 2x$
(ii) $f : \mathbb{R} \to \mathbb{R}, f(x) = e^{x^2}$

(b) Let $(A, \leq)$ be a poset. Consider the following two statements:
p : If $a \in A$ is a maximal element, then there is no $c \in A$ such that $a < c$.
q : If $a \in A$ is a maximal element, then for all $b \in A,\ b \leq a$.
Are the statements p and q equivalent? Justify your answer.

(5+5=10)

Question III (a) Check the validity of the following arguments:

(i)
$$\begin{array}{l} (p \Rightarrow q) \wedge (r \Rightarrow s) \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

(ii)
$$\begin{array}{l} (p \Rightarrow q) \wedge (r \Rightarrow s) \\ \sim q \vee \sim s \\ \hline \therefore \sim p \vee \sim r \end{array}$$

Contd.....P/2

(b) Prove that if a and b are elements in a bounded, distributive lattice and if a has a complement a', then

$$a \vee (a' \wedge b) = a \vee b,$$
$$a \wedge (a' \vee b) = a \wedge b.$$

(5+5=10)

Question IV (a) Define the following terms. Give a unique example for each of them. (i) Lattice; (ii) Bounded lattice; (iii) Distributive lattice; (iv) Complemented lattice.

(b) For $A = \mathbb{R} \times \mathbb{R}$, define relation R on A by $(x_1, y_1)R(x_2, y_2)$ if $x_1 = x_2$. Show that R is an equivalence relation on A. Describe geometrically the equivalence classes and partitions of A induced by R.

(5+5=10)

Part B

Question V (a) Find the generating function for the number of partitions of a positive integer n into **distinct** summands.

(b) Solve the recurrence relation $a_n - 3a_{n-} = 5(3^n)$, where $n \geq 1$ and $a_0 = 2$.

(5+5=10)

Question VI (a) Prove the following results about Eulerian circuits and paths in a connected graph G :

(i) There exists an Eulerian circuit in G if and only if each vertex in G has even degree.

(ii) There exists an Eulerian path in G if and only if G has exactly two vertices with odd degree. In this case, one of the vertex with odd degree is the starting vertex and the other vertex with odd degree is the end vertex of the Eulerian path.

(b) A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols, with at least 3 spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message?

(5+5=10)

Question VII (a) Define an Abelian Group. Characterize all the Abelian groups of order 4.

(b) Let $\mathbb{Z}_n$ be the set of modulo classes of the set of integers modulo n, where n is a natural number greater than 1. Show that $\mathbb{Z}_n$ is a ring with unity under the standard operations of addition and multiplication modulo n. For what values of n is $\mathbb{Z}_n$ a field? Justify.

(5+5=10)

x-x-x