Exam.Code:1015
Sub. Code: 35216

2055

M.E. (Mechanical Engineering)

Second Semester

MME-201: Continuum Mechanics

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, selecting atleast two questions from each Part. Make suitable assumptions where needed. All questions carry equal marks

x-x-x

PART A

Q.1

A. Given that $T_{ij} = 2\mu E_{ij} + \lambda E_{kk}\delta_{ij}$, show that (a) $T_{ij}E_{ij} = 2\mu E_{ij}E_{ij} + \lambda(E_{kk})^2$ and (b) $T_{ij}T_{ij} = 4\mu^2 E_{ij}E_{ij} + (E_{kk})^2(4\mu\lambda + 3\lambda^2)$. (5)

B. Use the identity $\varepsilon_{ijm}\varepsilon_{klm} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$ to show that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$. (5)

Q.2

A. Obtain the matrix for the tensor **T**, that transforms the base vectors as follows: $\mathbf{Te}_1 = 2\mathbf{e}_1 + \mathbf{e}_3$, $\mathbf{Te}_2 = \mathbf{e}_2 + 3\mathbf{e}_3$, $\mathbf{Te}_3 = -\mathbf{e}_1 + 3\mathbf{e}_2$. (3)

B. A tensor **T** has a matrix [T] given below. (a) Write the characteristic equation and find the principal values and their corresponding principal directions. (b) Find the principal scalar invariants. (c) If $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ are the principal directions, write $[\mathbf{T}]_{\mathbf{n}_i}$. (d) Could the following matrix [S] represent the same tensor **T** with respect to some basis? $[\mathbf{T}] = \begin{bmatrix} 5 & 4 & 0 \\ 4 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $[\mathbf{S}] = \begin{bmatrix} 7 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. (7)

Q.3

A. Prove the identity $\frac{d}{dt}(\mathbf{TS}) = \mathbf{T}\frac{d\mathbf{S}}{dt} + \frac{d\mathbf{T}}{dt}\mathbf{S}$ using the definition of derivative of a tensor. (5)

B. Consider the scalar field $\phi = x_1^2 + 3x_1x_2 + 2x_3$. (a) Find the unit vector normal to the surface of constant ϕ at the origin and at (1,0,1). (b) What is the maximum value of the directional derivative of ϕ at the origin? at (1,0,1)? (c) Evaluate $d\phi/d\mathbf{r}$ at the origin if $d\mathbf{r} = ds(\mathbf{e}_1 + \mathbf{e}_3)$. (5)

Q.4

Consider polar coordinates (r,θ), obtain components of ∇f, (10)

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Q.5

Consider the motion: $x_1 = \beta X_2^2 t^2 + X_1$, $x_2 = kX_2 t + X_2$, $x_3 = X_3$.

(a) At $t = 0$, the corners of a unit square are at $A(0, 0, 0)$, $B(0, 1, 0)$, $C(1, 1, 0)$ and $D(1, 0, 0)$. Sketch the deformed shape of the square at $t = 2$.

(b) Obtain the spatial description of the velocity field. (10)

(c) Obtain the spatial description of the acceleration field.

Q.6

Given the following right Cauchy-Green deformation tensor at a point

$$[\mathbf{C}] = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0.36 \end{bmatrix}.$$

(a) Find the stretch for the material elements that were in the direction of $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$.

(b) Find the stretch for the material element that was in the direction of $\mathbf{e}_1 + \mathbf{e}_2$.

(c) Find $\cos\theta$, where θ is the angle between $d\mathbf{x}^{(1)}$ and $d\mathbf{x}^{(2)}$ and where $d\mathbf{X}^{(1)} = dS_1\mathbf{e}_1$ and $d\mathbf{X}^{(2)} = dS_2\mathbf{e}_1$ deform into $d\mathbf{x}^{(1)} = ds_1\mathbf{m}$ and $d\mathbf{x}^{(2)} = ds_2\mathbf{n}$. (10)

Q.7

For any stress state $\mathbf{T}$ we define the deviatoric stress $\mathbf{S}$ to be $\mathbf{S} = \mathbf{T} - (T_{kk}/3)\mathbf{I}$, where T_{kk} is the first invariant of the stress tensor $\mathbf{T}$.

(a) Show that the first invariant of the deviatoric stress vanishes.

(b) Evaluate $\mathbf{S}$ for the stress tensor:

$$[\mathbf{T}] = 100\begin{bmatrix} 6 & 5 & -2 \\ 5 & 3 & 4 \\ -2 & 4 & 9 \end{bmatrix} kPa.$$

(c) Show that the principal directions of the stress tensor coincide with those of the deviatoric stress tensor. (10)

Q.8

The deformation of a body is described by

$$x_1 = 2X_1, \quad x_2 = 2X_2, \quad x_3 = 2X_3.$$

(a) For a unit cube with sides along the coordinate axes, what is its deformed volume? What is the deformed area of the $\mathbf{e}_1$ face of the cube?

(b) If the Cauchy stress tensor is given by

$$\begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix} Mpa,$$

calculate the first Piola-Kirchhoff stress tensor and the corresponding pseudo-stress vector for the plane whose undeformed plane is the $\mathbf{e}_1$-plane and compare it with the Cauchy stress vector on its deformed plane.

(c) Calculate the second Piola-Kirchhoff tensor and the corresponding pseudo-stress vector for the plane whose undeformed plane is the $\mathbf{e}_1$-plane. Also calculate the pseudo-differential force for the same plane. (10)

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