(i) Printed Pages : 4 Roll No. ....

(ii) Questions : 7 **Sub. Code : 3 3 2 9 1**

Exam. Code : 0 9 0 6

B.Engg. 1st Year (2nd Semester)

(2055)

DIFFERENTIAL EQUATIONS AND TRANSFORMS

Paper—ASM-201 (Common to All Stream)

Time Allowed : Three Hours] **[Maximum Marks : 50**

Note :— Attempt **five** questions in all, selecting **two** questions from each section. Question No. **1** is compulsory.

1. (a) Find the general solution of the following differential equation : 5×2=10

$$\frac{dy}{dx}+\frac{ax+hy+g}{hx+by+f}=0$$

(b) Using regrouping, solve the differential equation: $xdy = (x^5 + x^3y^2 + y)dx$.

(c) State and porve second shifting property of Laplace transforms.

(d) Form the partial differential equation by eliminating the arbitrary functions from $z = xf(x + y) + g(x + y)$.

(e) Define even and odd functions. Check whether the given periodic function is even or odd :

$$f(x)=x^2 \forall x\in(0,2\pi);\text{ with period } p=2\pi.$$

SECTION—A

2. (a) Find the general solution of the differential equation : 5

$$\left(D^2 - 4D + 4\right)y = 8x^2e^{2x}\sin 2x$$

(b) Solve the differential equation $\left(D^2 - 1\right)y = e^{-2x}\sin\left(e^{-x}\right)$ using method of variation of parameters. 5

3. (a) Solve the differential equation : 4

$$\left(D^2 + 16\right)y = e^{-2x} + \cos(4x)$$

(b) Evaluate the integral using Laplace transforms : 3

$$\int_0^{\infty}\int_0^{t} e^{-t}\frac{\sin u}{u}\,du\,dt$$

(c) Find the inverse Laplace transform of the function $\tan^{-1}\left(\frac{2}{s^2}\right)$. 3

4. (a) Using Laplace transform, solve the following differential equation : 4

$$ty'' + 2y' + ty = \sin t,\ y(0) = 1,\ y'(0) = 0$$

(b) Using Convolution theorem, find $L^{-1}\left[\frac{s^2}{s^4 - a^4}\right]$. 3

(c) If $L\left[f(t)\right] = \bar{f}(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^{\infty}\bar{f}(r)\,dr$ provided the integral exists. 3

SECTION—B

5. (a) Consider the even periodic function f(x) with period p = 4 defined below : 5

$$f(x)=\begin{cases}\pi x, & 0<x<1\\ \pi(2-x), & 1<x<2\end{cases}$$

Find its Fourier series representation and hence, find the sum of the series :

$$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots$$

(b) Express the function $f(x)=\begin{cases}1, & 0\le x<\pi\\ 0, & x\ge\pi\end{cases}$ as Fourier Cosine integral and hence evaluate

$$\int_0^\infty \frac{\cos(wx)\sin(\pi w)}{w}\,dw.$$ 5

6. (a) Find the general solution of the differential equation : 6

$$x(z+2a)p+(xz+2yz+2ay)q=z(z+a)$$

Also find the internal surface which passes through the curve : $y = 0, z^2 = 4ax$.

(b) Find the Fourier series expansion of the periodic function with period $p = 2\pi$: 4

$$f(x)=x^2, -\pi<x<\pi$$

7. Find the temperature u(x, t) in a laterally insulated bar of length L if the initial temperature is f(x) and the ends are kept at 0°C. The temperature u(x, t) satisfies the following differential equation : 10

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$