**Exam.Code:0918**
**Sub. Code: 33443**

**2015**

**B.E. (Computer Science and Engineering)**
**Sixth Semester**
**CS-602: Linear Algebra and Probability Theory**

**Time allowed: 3 Hours** **Max. Marks: 50**

***NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. Use of a simple calculator and statistical tables is allowed. All questions carry equal marks.***

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1. (a) Define a vector space and list its properties. Determine whether the set of all $(2 \times 2)$ matrices with determinant 1 form a vector space?

   (b) Define diagonalization of a matrix and explain its significance. Diagonalize the matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, if possible.

   (c) Give an example of a linear transformation that is not one-to-one. Explain briefly how linear transformations are used in computer graphics and image processing.

   (d) Define a linear combination of random variables. Provide the formula for means and variance of a linear combination of random variables. Explain how means and variances change when combining random variables?

   (e) Define covariance for two-dimensional random variables. What does positive, negative and zero covariance indicate about two random variables? Can correlation be 0 if covariance is not 0?

## SECTION-A

2. (a) Solve the homogeneous system: $AX = 0$, where $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 3 & 2 \end{bmatrix}$. Find the rank (A) and nullity(A). $(3 + 4 + 3)$

   (b) Prove that the set of all upper triangular matrices of order 2 over R is a vector space with standard matrix addition and scalar multiplication.

   (c) Find conditions on $x, y, z$ so that the vector $s = (x, y, z) \in R^3$ belongs to the space generated by $u = (1, -1, 2), v = (2, 1, 0)$ and $w = (0, 3, -4)$.

3. (a) Find the eigenvalues and eigenvectors for the matrix: $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   (b) Define similar matrices and their properties. Examine whether A is similar to B, where $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 5 \\ -2 & 0 \end{bmatrix}$. $(4 + 3 + 3)$

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(c) Examine whether the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ is diagonalizable or not? If yes, diagonalize it.

4. (a) State rank-nullity theorem. Consider a linear transformation $T: R^3 \rightarrow R^2$ defined by the matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. Find the rank and nullity of the linear transformation and verify rank-nullity theorem.

(b) Find the matrix representation of the linear operator relative to given basis of $R^3$, where $T: R^3 \rightarrow R^3$ is defined by $T(x, y, z) = \{2z, x - 2y, x + 2y\}$ and basis $B = \{(1, 2, 1), (1, 1, 1), (1, 1, 0)\}$.

## SECTION-B

5. (a) Describe the sample space for the experiment of flipping three coins sequentially. $(2 + 4 + 4)$

(b) An urn contains nine balls, two of which are red, three blue and four black. Three balls are drawn from the urn at random. What is the probability that the three balls are of the same colour?

(c) A factory produces light bulbs, and 2% of the bulbs are defective. If a bulb is defective, the probability of it passing the quality test is 0.2. If a bulb passes the test, what is the probability that it is defective?

6. (a) What is a probability distribution? Explain the difference between discrete and continuous probability distribution with suitable examples..

(b) Suppose you have a random variable with mean 50 and variance 25. Use the Chebyshev inequality to find an upper bound on the probability that X deviates from its mean by more than 10 units. $(3 + 4 + 3)$

(c) Prove that Poisson distribution is the limiting case of binomial distribution for very large trials with very small probability.

7. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(b) The joint probability distribution function of the continuous random variable X and Y is given by $f(x, y) = \begin{cases} 2\,x\,e^{-y}, 0 < x < 1, y > 0, \\ 0, \text{elsewhere} \end{cases}$. Find the distribution of $X + Y$.

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