

2015

B.E. (Mechanical Engineering)

Fourth Semester

MEC-406: Numerical Analysis

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Part. Each question carries equal marks. Use of a simple calculator is allowed.

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1. (a) Define significant figures and explain their importance in scientific measurements. Describe the rules for determining the number of significant figures in a given value.
- (b) Discuss the convergence criteria for root finding method. How do you determine when to stop iterating?
- (c) Compare and contrast the Gauss elimination method with LU decomposition. In what situations would you prefer one method over the other?
- (d) Explain Simpson's rules for numerical integration. How does it differ from the trapezoidal rule, and under what conditions is it more accurate?
- (e) Discuss the finite difference method for solving partial differential equations.

PART-A

2. (a) What is the importance of series approximation in numerical analysis? What is truncation error in series approximation? How can it be minimized?
- (b) Given $U = xy + yz + zx$, find the relative percentage error in the computation of U at $x = 2.104, y = 1.935, z = 0.845$. (3 + 3 + 4)
- (c) Apply fixed point iteration method to approximate the value $\sqrt{42}$ correct to three decimal places.
3. (a) Define the condition number of a linear system of equations. Explain how it is computed and what it signifies about the stability of the system. Calculate the condition number of the matrix : $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$, using norm 2.
- (b) Solve the following linear system using the Gauss-Seidel iteration method:

$$x + y + 3z = 5; 10x + y + z = 12; 2x + 10y + z = 13.$$

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(2)

4. (a) Suppose you have a dataset with two predictor variables X_1, X_2 and one outcome variable Y :

X_1	10	20	30	40	50
X_2	5	10	15	20	25
Y	100	200	300	400	500

Build a multiple linear regression model to predict Y based on X_1 and X_2 . Also estimate the coefficients for each predictor variable.

- (b) Explain how Lagrange's formula can be used to express the rational function as a sum of partial functions. Express $f(x) = \frac{x^2+x-3}{x^3-2x^2-x+2}$ as a sum of partial fractions.

PART-B

5. (a) Determine the volume of the solid obtained by rotating the region bounded by $y = e^x$, $x = 0$, $x = 2$ and the x -axis around the x -axis. Use Simpson one-third rule with nine nodes.

- (b) Using Richardson extrapolation formula to evaluate $\frac{dy}{dx}(1.6)$ for the following data:

x	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4
y	1.3	1.7	2.3	3.2	4.7	6.2	8.1	9.2	9.8

6. (a) Use Runge-Kutta method to obtain $y(1.1)$ given that $y(1) = 1.2$ and

$$\frac{dy}{dx} = 3x + y^2.$$

- (b) Solve the BVP: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$, $y(0) = 1$, $y(1) = 0$, using the second order finite difference method with $h = \frac{1}{4}$.

7. Solve the Laplace equation: $u_{xx} + u_{yy} = 0$ in $0 \leq x \leq 4$, $0 \leq y \leq 4$, given that

$$u(0, y) = 0, u(4, y) = 8 + 2y, u(x, 0) = \frac{x^2}{2} \text{ and } u(x, 4) = x^2. \text{ Take } h = k = 1 \text{ and}$$

obtain the result correct to two decimal places.