Exam. Code: 0906
Sub. Code: 33291

2015

B.E., Second Semester

ASM-201: Differential Equations and Transforms

(Common to all streams)

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt <u>five</u> questions in all, including Question No. I which is compulsory and selecting two questions from each Part. Use of non programmable calculator is allowed***

x-x-x

1. (a) Find the general solution of the differential equation: $(x-y)^2\frac{dy}{dx} = a^2$

(b) Solve the differential equation: $\frac{dy}{dx} + y = \frac{1}{1+e^{2x}}$.

(c) Find the Laplace transform of the following function

$$f(t) = \begin{cases} \sin 2t & \text{if } 0 \le t \le \pi \\ 0 & \text{if } t > \pi \end{cases}$$

(d) Define a periodic function and its fundamental period. Also find the fundamental period of the periodic function $f(x) = \cos(\pi x)$.

(e) Formulate the partial differential equation by eliminating the arbitrary function: $z = f\left(\frac{xy}{z}\right)$ $(5 \times 2 = 10)$

PART A

2. (a) Find the general solution of the following differential equation: (5)

$$(D^6 + 3D^4 + 3D^2 + 1)y = 3e^{2x} + 4\sin x$$

(b) Solve the following differential equation by the method of variation of parameters: (5)

$$(D^2 + 2D + 2)y = 4e^{-x}\sec^3 x$$

3. (a) Solve the given differential equation: (4)

$$(D^2 + D + 1)y = e^x$$

(b) Find the inverse Laplace transform of $\frac{1}{s}e^{-1/\sqrt{s}}$. (3)

(c) Prove that $\int_{t=0}^{\infty}\int_{u=0}^{t} \frac{e^{-t}\sin u}{u} du\, dt = \frac{\pi}{4}$ (3)

4. (a) State and prove convolution theorem for Laplace transform. (5)

(b) Solve the following differential equation using Laplace transforms: (5)

$$y''' - 3y'' + 3y' - y = t^2e^t,\ y(0) = 1,\ y'(0) = 0,\ y''(0) = -2$$

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PART B

5. (a) Find the fourier series of the function (5)

$$f(x) = \begin{cases} x(x+\pi), & \text{if } \; -\pi < x < 0, \\ x(-x+\pi) & \text{if } \; 0 < x < \pi \end{cases}$$

(b) Find the Fourier cosine and sine integral represntations of the following function: (5)

$$f(x) = \begin{cases} e^{-x}, & \text{if } \; 0 < x < a, \\ 0 & \text{if } \; x > a \end{cases}$$

6. (a) Find the general integral of the equation for the following partial differential equation

$$(x-y)p + (y-x-z)q = z$$

and the particular solution through the circle $z = 1,\ x^2 + y^2 = 1.$ (5)

(b) Find the general integral of the linear partial differential equations: (5)

$$(y+zx)p - (x+yz)q = x^2 - y^2$$

7. Find D'Alembert's solution of the one dimensional wave equation (10)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where $u(x,t)$ represents the deflection in an elastic string of length L. Given that initial deflection in the string is $f(x)$ and intial velocity is $g(x)$.

x-x-x