

B.E. (Electronics and Communication Engineering)
Third Semester
MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section. All questions carry equal marks.

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1. (a) Give any two practical situations where the problems are modeled by system of linear equations. Give one example of a non-homogeneous system whose solution set is empty.
- (b) Define subspace of a vector space. Examine whether the set of all 2×2 singular matrices is a subspace of M_{22} or not? Justify.
- (c) Define basis of a vector space. Let $V = \{0\}$ be a zero vector space over \mathbb{R} . State basis for V with proper justification.
- (d) Define limit and continuity of a complex function at a point. How are they different from their real counterparts?
- (e) Define fixed point of a mapping. Hence, find the same for $w = \frac{1}{z}$.

SECTION-A

2. (a) Discuss the advantages of representing a system of linear equations in triangular form for computational purposes. Analyze the consistency of the following system using row echelon form: (3 + 4 + 3)

$$2x + y + 3z = 7; 4x - y + z = 3; 6x - 3y + 2z = 5.$$

- (b) Define vector space. Verify all vector space axioms for the set P_2 of polynomials of degree less than or equal to two. Will the polynomials of degree two only form a vector space? Justify.
- (c) State and prove that any two bases of a finite dimensional vector space have the same number of elements. Give an example of \mathbb{R}^3 .
3. (a) Let $W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + y - z = 0 \text{ and } y + z + w = 0\}$. Prove that W is a subspace of \mathbb{R}^4 . Find the basis and dimension of W . Extend the basis of W to a basis for the entire space \mathbb{R}^4 .

- (b) Examine whether the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is diagonalizable or not? If yes,

the diagonalize it. Using the diagonalization, compute the determinant of A and the trace of A^3 .

(2)

4. (a) Let $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ defined below be a linear transformation. Find the basis and dimension of its range space and null space. Also, verify rank-nullity theorem.

$$T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t).$$

- (b) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (2x + y, x - 2y)$. Consider two bases: $B_1 = \{(1, 2), (3, 1)\}$ and $B_2 = \{(1, 1), (1, -1)\}$. Find the matrix of T with respect to the basis B_1 . Find the transition matrix from B_1 to B_2 . Use these results to find the matrix of T with respect to the basis of B_2 .

SECTION-B

5. (a) Find the real and imaginary parts of $\cosh z$ and $\sinh z$. Prove that both of them are unbounded functions. (4 + 3 + 3)

(b) Test differentiability and analyticity of $f(z) = \bar{z}$ (complex conjugate of z).

(c) Find the general value of $\log(1 + i) + \log(1 - i)$.

6. (a) Find the Taylor's and Laurent's series which represents the function

$$f(z) = \frac{z^2 - 1}{(z + 2)(z + 3)} \text{ when (i) } |z| < 2, \text{ (ii) } 2 < |z| < 3.$$

(b) Find and classify all the isolated singularities including their order for the function $f(z) = \frac{\cos(\pi z)}{z^2(z-1)^3}$. Also, determine the residue at each pole.

7. (a) Using contour integration, evaluate the integral: $\int_0^\pi \frac{(1+2\cos\theta)}{4\cos\theta+5} d\theta$.

(b) Discuss the mapping $w = e^z$. Find the image of the lines $x = \text{constant}$ and $y = \text{constant}$.