

B.E. (Computer Science and Engineering)  
Third Semester  
CS-303: Discrete Structures

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.

x-x-x

1. Briefly explain the following with example:

- Irreflexive and Anti-symmetric relations
- Bipartite graph and K-Regular graph
- Monoid and Semigroup
- Chain and lattice
- Quantifiers

(5x2=10)

Section-A

2. Prove the validity of following arguments without using truth tables.

- $p \vee q, \neg p \vdash q$
- $p \rightarrow (q \vee r), (s \wedge t) \rightarrow q, (q \vee r) \rightarrow (s \wedge t) \vdash p \rightarrow q$
- $p, p \rightarrow q, q \rightarrow r \vdash r$
- If you get passing marks in the exam, then you do not get prize. If you get highest marks in the exam, then you get prize. You get passing marks or you get highest marks in the exam. Therefore, you get prize.
- You go to market or you go to play. You do not go to market or you go to home. Therefore, either you go to play or you go to home.

(5x2=10)

3. (a) Consider the function  $f: \mathbb{N} \rightarrow \mathbb{N}$ , where  $\mathbb{N}$  is the set of natural numbers including zero defined by  $f(n) = n^2 + 2$ . Check whether the function  $f$  is (i) one-one (ii) onto. (4)

(b) Let  $R$  be a binary relation on  $A$  such that  $(a, b) \in R$ , if book 'a' costs more and contains fewer pages than book 'b'. Is  $R$  an Equivalence Relation or a Partial Order Relation? Justify. (3)

(c) Prove that there is no largest integer that is a multiple of 5 using a suitable method of proof. (3)

4. (a) Consider the universal set  $S = \{1, 2, 3, 4, \dots, 10\}$  and the subsets  $A = \{2, 4, 6\}$ ,  $B = \{1, 5, 8, 9\}$ ,  $C = \{3, 7, 10\}$ . List the non-empty minsets generated by  $A$ ,  $B$  and  $C$ . Do the minsets form a partition of  $U$ ? (3)

(b) Prove that  $\forall x(P(x) \rightarrow Q(x)) \wedge \sim Q(y) \Rightarrow \sim \forall xP(x)$  (3)

(c) Let  $D_{100}$  is a set containing all the divisors of 100. Let the relation  $<$  be the relation  $|$  (divides) be a partial ordering on  $D_{100}$ .

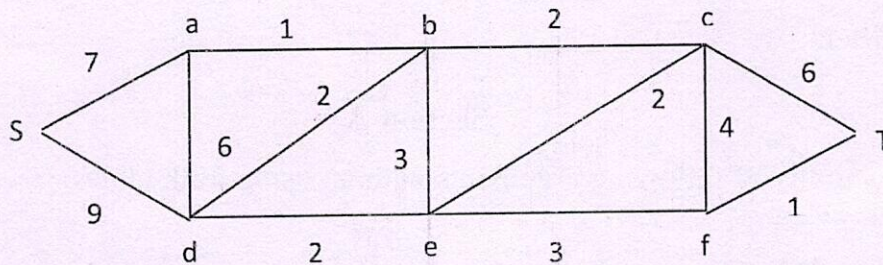
- Draw the Hasse diagram of the given poset.
- Determine the glb of  $\{5, 25\}$  and  $\{4, 20, 50, 100\}$
- Determine the lub of  $\{5, 25\}$  and  $\{4, 20, 50, 100\}$

(4)

(2)

**Section-B**

5. (a) Solve the recurrence relation  $a_r - 2a_{r-1} + a_{r-2} = 2^r$ ,  $r \geq 2$ , by the method of generating function satisfying the boundary conditions  $a_0 = 2, a_1 = 1$ . (6)
- (b) What do you understand by Isomorphic and Homeomorphic graphs? Explain using examples. (4)
6. (a) Consider  $a \in \mathbb{R}$  as a constant real number. Assume  $G = \{a^n : n \in \mathbb{Z}\}$ . Prove that  $G$  is an abelian group under usual multiplication. (5)
- (b) Discuss the Breadth-First Traversal technique using the given graph. (5)



7. (a) Differentiate between a ring and a field. (3)
- (b) Is  $E$ , the set of even integers a commutative ring without unity? Justify your answer. (3)
- (c) Find the number of ways in which 6 boys and 6 girls can be arranged in a row under the following conditions:
- I. All boys are to be seated together and all girls are to be seated together.
- II. Boys and girls take their seats alternately. (4)