101

Exam.Code:0921
Sub. Code: 33521

2125

B. E. (Information Technology)
Third Semester
ASM-301: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. 1 (section-A) which is compulsory and selecting two questions each from Section B- C.

x-x-x

Section - A

1. Answer the following:

a) State rank-nullity theorem.

b) Define change-of-Basis matrix.

c) Compute eigenvalues of the matrix $\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$.

d) State the Central Limit Theorem and Chebyshev's inequality.

e) Find the probability of getting a total of 7 at least once in three tosses of a pair of fair dice.

$(5 \times 2 = 10)$

Section - B

2. a) Let $A = \begin{pmatrix} 1 & -2 & 3 & 1 & 2 \\ 1 & 4 & 1 & -3 & 3 \\ 2 & 5 & 9 & -2 & 8 \end{pmatrix}$. Use Gauss–Jordan method to find the row canonical form of A.

b) Let W be the subspace spanned by the vectors: $u_1 = (1, 2, -1, 3)$, $u_2 = (2, 4, -2, 6)$, $u_3 = (1, 3, 2, 6)$, $u_4 = (1, 4, 5, 18)$, $u_5 = (2, 7, 3, 9)$. Find a subset of these vectors that forms a basis of W.

c) Determine whether the vectors $(1, 1, 2)$, $(2, 3, 1)$, $(4, 5, 3)$ in $\mathbb{R}^3$ are linearly independent.

$(04 + 04 + 02)$

3. a) Find a homogeneous system whose solution is spanned by following set of vectors:

$$(1, -2, 0, 3, 1),\ (2, -3, 2, 5, -3),\ (1, 2, 1, 2, -2).$$

b) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (2x+y-2z,\ 2x+3y-4z+3)$. Find all eigenvalues of T and determine whether T is diagonalizable. (05+05)

4. a) Let $G : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $G(x, y, z) = (2x+3y-3,\ y-z+2)$. Find the matrix A representing G relative to the basis $S = \{(1, 1, 0),\ (1, 2, 3),\ (1, 3, 5)\}$. Verify that $[G]_S[v]_S = [G(v)]_S$ for $v = (a, b, c)$.

b) Find the characteristic polynomial of the matrix $A = \begin{pmatrix} 2 & 5 & 1 & 1 \\ 1 & 4 & 2 & 2 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & 2 & 3 \end{pmatrix}$.

c) Find $F(a, b)$, where the linear map $F : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by: $F(1, 2) = (3, -1), F(0, 1) = (2, -1)$.

$(04 + 04 + 02)$

P.T.O.

Section - C

5. a) The probability that a man will hit a target is If he shoots at the target until he hits it for the first time, find the probability that it will take him 5 shots to hit the target.

b) An urn holds 5 white and 3 black marbles. If 2 marbles are to be drawn at random without replacement and X denotes the number of white marbles, find the probability distribution for X. Let $Y = X^2$. Determine (i) the distribution g of Y, (ii) the joint distribution h of X and Y, (iii) covariance(X, Y) and correlation(X, Y) (04 + 06)

6. a) The joint density function of two continuous random variable X and Y is

$$f(x,y) = \begin{cases} cxy & 0 < x < 4,\ 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Find the value of the constant c. (b) Find the marginal distribution of X and Y.

b) A random variable X has density function given by

$$f(x,y) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find (a) the moment generating function, (b) the first four moments about the origin

(05 + 05)

7. a) Prove that the mean and variance of a binomially distributed random variable are, respectively, $\mu = np$ and $\sigma^2 = npq$.

b) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals, (a) exactly 3, (b) more than 2, individuals will suffer a bad reaction. (05 + 05)

x-x-x