Exam.Code:1014
Sub. Code: 35201

2125
M.E. (Mechanical Engineering)
First Semester
MME-101: Advanced Engineering Mathematics

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, selecting atleast two questions from each Section. Use of Simple calculator is allowed. All questions carry equal marks.

x-x-x

SECTION-A

1. (a) Discuss the advantages and limitations of the power series method for solving second-order differential equations. When this method inapplicable?

   (b) Find the power series solution of the differential equation:

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0 \text{ about } x = 0.$$

2. (a) Derive the Rodrigues formula for Legendre polynomials from Legendre's differential equation.

   (b) Use the Frobenius method to obtain a series solution about $x = 0$ for

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

3. (a) Obtain the recurrence relation for Bessel's functions of the first kind:

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x}J_n(x).$$

   (b) Reduce the equation: $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{9}{4x^2}\right)y = 0$ to the standard Bessel form and identify its order. Also ind its solution.

4. (a) For the boundary value problem: $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\pi) = 0$, find the eigenvalues and normalized eigenfunctions. Verify their orthogonality.

   (b) Discuss the concept of eigenfunction expansion in terms of orthogonal functions. Explain how a function f(x) can be represented as a series of eigenfunctions of a Sturm-Liouville problem.

SECTION-B

5. (a) Explain how higher order differential equations can be reduced to a system of first-order equations. Illustrate with an example.

(b) Solve the system: $\frac{dy}{dx} = z; \frac{dz}{dx} = -y$ for $x = 0.3$ using the Runge-Kutta method, given $y(0) = 0, z(0) = 1$.

6. (a) Solve the boundary value problem: $\frac{d^2y}{dx^2} = y + x, y(0) = 0, y(1) = 1$using the finite difference method with step size $h = 0.1$.

(b) Derive the five-point finite difference formula for Laplace's equation.

7. Compare elliptic, parabolic and hyperbolic partial differential equations in terms of their finite difference approximations and physical meaning.

8. Solve numerically $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ with the following boundary conditions $u(0,t) = 0, u(4,t) = 0, t > 0$and the initial conditions $\frac{\partial u}{\partial t}(x,0) = 0, u(x,0) = 4x - x^2$ taking $h = 1$for four time steps.

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