CSE

Exam.Code:0905
Sub. Code: 33265

2125
B.E., First Semester
ASM-101: Calculus
(Common to all streams)

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.

x-x-x

Question I (a) Check the absolute and conditional convergence of the alternating series:

$$\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{\log n}$$

(b) Show that the series $\sum_{n=1}^{\infty}\frac{1}{n(1+\ln^2 n)}$ satisfies the hypothesis of the integral test. Hence show that that this series converges.

(c) Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$, and the x-axis.

(d) Define curvature and torsion for a space curve.

(e) Find the length of the graph of $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \le x \le 4$.

$(2 \times 5 = 10)$

Part A

Question II Discuss the convergence of the following series:

(i) $\sum_{n=1}^{\infty}\frac{1}{n}$ (ii) $\sum_{n=1}^{\infty}\left(\frac{1}{2^{1/n}} - \frac{1}{2^{1/(n+1)}}\right)$ (iii) $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$ (iv) $\sum_{n=1}^{\infty}\frac{8\tan^{-1}n}{1+n^2}$

(v) $\sum_{n=1}^{\infty}\frac{(n+3)!}{3!\ n!\ 3^n}$

(10)

Question III (a) Obtain the fourth degree Taylor's polynomial approximation to $f(x) = e^{2x}$ about $x = 0$. Find the maximum error when $0 \le x \le 0.5$

(b) If $w = f(u, v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$ and if $u = (x^2 - y^2)/2$ and $v = xy$, then find the value of $w_{xx} + w_{yy}$.

(5+5=10)

Question IV (a) A standard 12-fl oz can of soda is essentially a cylinder of radius $r = 1$ in. and height $h = 5$ in.

(i) At these dimensions, how sensitive is the can's volume to a small change in radius versus

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a small change in height?
(ii) Could you design a soda can that appears to hold more soda but in fact holds the same 12-ft oz? What might its dimensions be?

(b) Find the absolute maxima and minima of the function: $f(x, y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0, y = 0, y + 2x = 2$ in the first quadrant.

(5+5=10)

Part B

Question V (a) Find using double integral the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$, and the x-axis.

(b) Find the volumes using triple integrals of the region between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$

(5+5=10)

Question VI (a) Find curvature (κ) and torsion (τ) for the following curve:

$$\mathbf{r(t)} = \ln(\sec t)\ \hat{\imath} + t\ \hat{\jmath}, -\pi/2 < t < \pi/2$$

(b) Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. In which direction does f change most rapidly at P_0, and what are the rates of change in these directions?

(5+5=10)

Question VII (a) Find the work done by force field $\vec{F} = z\,\hat{\imath} + x\,\hat{\jmath} + y\,\hat{k}$ over the curve $\vec{r}(t) = \sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + t\,\hat{k},\ 0 \leq t \leq 2\pi$ in the direction of increasing t.

(b) Use divergence theorem to calculate the outward flux of $\mathbf{F}$ across the boundary of the region D: The region inside the solid cylinder $x^2 + y^2 \leq 4$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$ where $\mathbf{F} = y\ \hat{\imath} + xy\ \hat{\jmath} - z\hat{k}$

(5+5=10)