Exam.Code:0918
Sub. Code: 6365

2054

B.E. (Computer Science and Engineering)

Sixth Semester

CS-602: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. Use of a simple calculator and statistical tables is allowed. All questions carry equal marks.***

x-x-x

1. (a) Define a vector space with suitable examples. Determine whether the set of all functions f(x) that satisfy $f''(x) = 0$ form a vector space? Justify.

   (b) State and prove the Cayley-Hamilton theorem for a two by two matrix. Also, write down its applications.

   (c) Define a linear transformation between two vector spaces. Explain the properties that a function must satisfy to be considered a linear transformation.

   (d) Define joint and marginal probability distributions. What is the relationship between them? Also, explain the difference between them.

   (e) Define the moment generating function of a random variable. Explain their importance in probability theory.

SECTION-A

2. (a) Examine whether the given system of equations is consistent or not. If consistent, solve it: $3x + y + z = 2;\ x - 3y + 2z = 1;\ 7x - y + 4z = 5$.

   (b) Define linear combination of vectors. Prove that the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ is a linear combination of matrices: $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $M_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

   (c) Examine whether the set of vectors $S = \{(1, 1, 1, 1), (1, 2, 1, 2)\}$ is linearly independent or dependent. Expand this set to be a basis of R^4. (3 + 4 + 3)

3. (a) Prove that the matrix: $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence, find the matrix P such that $P^{-1}AP$ is diagonalizable. (5 + 5)

   (b) Prove that the linear transformation $T: R^2 \to R^2$ defined by: $T(x, y) = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$ is a vector space isomorphism.

4. (a) Find a linear transformation, which maps $(1, 1, 1), (1, 1, 0), (1, 0, 0)$ in R^3 to $(3, 1,), (3, 1), (3, 1)$ in R^3.

   (b) Let T be a linear operator on R^3 defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$. Find the matrix of T relative to the basis $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$.

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SECTION-B

5. Consider the experiment of rolling two six-sided dice. Describe the sample space for this experiment. (2 + 4 + 4)

(b) In a town, 60% of the population owns a cat and 40% owns a dog. Among cat owners, 25% also own a dog. What is the probability that a randomly selected person owns a dog given that they own a cat?

(c) State the law of total probability and explain its significance in probability theory.

6. (a) Define expected value and discuss its interpretation in terms of long term average outcomes. (2 + 5 + 3)

(b) Determine the discrete probability distribution, expectation, variance, standard deviation of a discrete random variable X, which denotes the minimum of the two numbers when a pair of fair dice is thrown once.

(c) If X is uniformly distributed in $-2 \leq x \leq 2$, find (i) $P(X < 1)$, (ii) $P|X - 1| \geq \frac{1}{2}$.

7. (a) In a distribution which is exactly normal, 12% of the items are under 30 and 85% are under 60. Find the mean and standard deviation of the distribution.

(b) The joint density function of two random variables X and Y is

$$f_{X,Y}(x, y) = \begin{cases} \frac{(x+y)^2}{40}, = 1 < x < 1 \; and \; -3 < y < 3 \\ 0, elsewhere. \end{cases}$$

Find (i) the variances of X and Y, (ii) the correlation coefficient.

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