Exam. Code: 0940
Sub. Code: 6716

2054

B.E. (Mechanical Engineering)
Fourth Semester
MEC-406: Numerical Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Unit. Each question carries equal marks. Use of a simple calculator is allowed.***

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1. Attempt the following: -

(a) Define total error and explain how it arises from round-off error, truncation error, and other sources. Discuss the procedures for minimizing total numerical error in computational algorithms.

(b) Compare and contrast the Secant method with Newton's method. In what situations would you prefer one over the other?

(c) Explain the concept of matrix inversion using Gauss elimination. Provide an example and discuss any limitations of this approach.

(d) Define numerical differentiation. What are its limitations? Describe Richardson extrapolation in the context of numerical differentiation.

(e) Explain the Crank-Nicoloson method for solving parabolic PDEs.

UNIT-I

2. (a) Explain how absolute error changes according to multiplication and division of two approximate numbers. (3)

(b) If $u(x, y, z) = 10\,x^3y^2z^2$ and error in x, y, z are respectively, $0.03, 0.01, 0.02$ at $x = 2, y = 2, z = 1$, calculate the relative and percentage errors in the calculation of it. (3)

(c) Find the root of the equation: $\cos x - x = 0$ correct to three decimal places using the bisection method. How many iterations are required to achieve this much accuracy? (4)

3. (a) Solve the following linear system by the LU decomposition method: $2x + y + 4z = 12;\ 8x + 3y + 2z = 20, 4x + 11y + z = 33$

(b) Explain the Gauss-Seidel iteration method for solving systems of linear equations. Solve the following linear system:

$$x - y + 5z = 7;\ x + 4y - z = 6;\ 6x + y + z = 20.$$

P.T.O.

4. (a) Suppose you have the following datasets representing the height y of a bouncing ball over time x:

x	0	1	2	3	4	5
y	1	3	5	3	1	3

Suppose that the height of the ball follows a sinusoidal pattern due to its bouncing motion. Fit the best sinusoidal function to this data.

(b) Using the following table, find f(x) as a polynomial in the powers of $(x-6)$:

x	-1	0	2	3	7	10
f(x)	-11	1	1	1	141	561

UNIT-II

5. (a) Evaluate $\int_1^2 \sqrt{1+\cos^2 x}\, dx$ with the aid of trapezoidal and Simpson one-third rule.

(b) Use Richardson extrapolation to estimate the first derivative of $y = \cos x$ at $x = \frac{\pi}{4}$ using the step sizes of $h_1 = \frac{\pi}{3}$ and $h_2 = \frac{\pi}{6}$.

6. (a) Using Runge-Kutta fourth order method, solve y(0.2) given that

$$\frac{dy}{dx} = x\,y + y^2, y(0) = 1.$$

(b) Solve the BVP $\frac{d^2y}{dx^2} - y = 0$ with $\frac{dy}{dx}(0) = 0$ and $y(1) = 1$.

7. Using the Bender-Schmidt formula to solve the heat conduction problem:
$\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(x,0) = 4x - x^2$ and $u(0,t) = u(4,t) = 0$.

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