Exam. Code: 0922
Sub. Code: 6482

2054

B.E. (Information Technology) Fourth Semester

ASM-401: Discrete Structures

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Part.***

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Question I (a) What is wrong with the following argument: Let R be a relation which is symmetric and transitive. Therefore $aRb \Rightarrow bRa$ and $aRb,\ bRa \Rightarrow aRa$ which implies R is reflexive.

(b) In how many ways can we distribute seven apples and six oranges among four children so that each child receives at least one apple?

(c) What do you mean by a derangement. How many derangements of 1, 2, 3, 4 are possible?

(d) Draw a planar representation of the bipartite graph $K_{2,3}$. Is $K_{3,3}$ planar? Explain graphically.

(e) Define the characteristic of a field. What can be the possible values of the characteristic of a field?

(5×2=10)

Part A

Question II (a) Let $A = \{1, 2, 3, ..., 9\}$ and let $\sim$ be the relation on $A \times A$ defined by

$$(a, b) \sim (c, d) \quad \text{if} \quad a + d = b + c$$

(i) Prove that $\sim$ is an equivalence relation.
(ii) Find $[(2, 5)]$, the equivalence class of $(2, 5)$.

(b) Let $A = \{1, 2\},\ B = \{m, n, p\},\ C = \{3, 4\}$. Define the relations $\Re_1 \subseteq A \times B,\ \Re_2 \subseteq B \times C,\ \Re_3 \subseteq B \times C$ by $\Re_1 = \{(1, m), (1, n), (1, p)\},\ \Re_2 = \{(m, 3), (m, 4), (p, 4)\}$ and $\Re_3 = \{(m, 3), (m, 4), (p, 3)\}$. Determine $\Re_1 \circ (\Re_2 \cap \Re_3)$ and $(\Re_1 \circ \Re_2) \cap (\Re_1 \circ \Re_3)$.

(5+5=10)

Question III (a) Find, if they exist, all upper bounds, all lower bounds, the least upper bound, the greatest lower bound of the subset B of the partially ordered set $(A, \leq)$ for the following:

(i) $A = \mathbb{R}$, the set of real numbers and $\leq$ denotes the usual partial order, $B = \{x \mid x \text{ is a real number and } 1 < x < 2\}$.

(ii) $A = \mathbb{R}$, the set of real numbers and $\leq$ denotes the usual partial order, $B = \{x \mid x \text{ is a real number and } 1 \leq x < 2\}$.

(iii) A is the set of 2×2 Boolean matrices and $\leq$ denotes the relation R with MRN if and only if $m_{ij} \leq n_{ij}$ for $1 \leq i \leq 2,\ 1 \leq j \leq 2$; $B = \left\{ \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

(b) Let $A = \mathbb{N}$ be the set of natural numbers. Let $R = \{(a, b) \in A \times A \mid a \text{ divides } b\}$. Is R symmetric, asymmetric or anti-symmetric? What if $A = \mathbb{Z} \backslash \{0\}$ is the set of non-zero integers.

(5+5=10)

P.T.O.

Question IV (a) Prove the following version of the Pigeon Hole Principle: Suppose there are n pigeon holes. To ensure that at least one pigeon hole contains at least k pigeons, the total number of pigeons must be at least $m = n(k-1)+1$.

(b) Write the following argument in symbolic form. Then establish the validity of the argument or give a counter example to show that it is invalid:

If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night then Carmela will be mad. If either Helen or Carmela gets mad, then Veronica (their attorney) will be notified. Veronica has not heard from either of these two clients. Consequently, Dominic didn't make it to the racetrack and Ralph didn't play cards all night.

(5+5=10)

Part B

Question V (a) Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$, $n \geq 0$, $a_0 = 0$, $a_1 = 1$, $a_2 = 2$.

(b) Find the number of ways of placing 20 similar balls into 6 numbered boxes so that the first box contains any number of balls between 1 and 5 inclusive and the other 5 boxes must contain 2 or more balls each. Use generating functions.

(5+5=10)

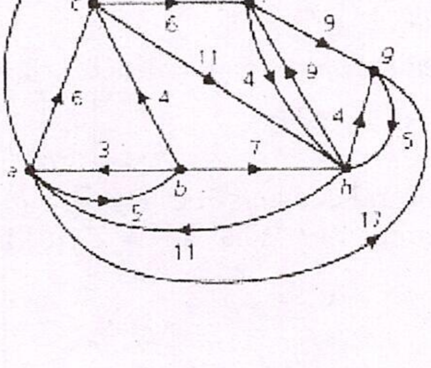
Question VI (a) Define an Eulerian graph. What is the necessary and sufficient condition for a connected graph to be Eulerian? State and prove.

(b) Explain with help of examples, the following terms in respect of a graph:
(i) Degree of a vertex
(ii) Discrete graph
(iii) Complete graph
(iv) Subgraph of a graph
(v) Connected graph

(5+5=10)

Question VII (a) Let $\mathbb{Z}_n$ be the set of modulo classes of the set of integers modulo n, where n is a natural number greater than 1. Show that $\mathbb{Z}_n$ is a ring with unity under the standard operations of addition and multiplication modulo n. For what values of n is $\mathbb{Z}_n$ a field? Justify.

(b) Using Dijkstra's algorithm to the weighted graph below, find the shortest distance from the vertex c to each of the other five vertices in G.



(5+5=10)

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