Exam.Code:1015
Sub. Code: 7443

2063
M.E. (Mechanical Engineering)
Second Semester
MME-201: Continuum Mechanics

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, selecting atleast two questions from each Section. Use usual notations and symbols for derivations. Assume suitably missing data if any. All questions carry equal marks

x-x-x

Section A

Q.1 Let $\mathbf{T}$ and $\mathbf{S}$ be any two tensors. Show that (a) $\mathbf{T}^T$ is a tensor. (b) $\mathbf{T}^T + \mathbf{S}^T = (\mathbf{T}+\mathbf{S})^T$, and (c) $(\mathbf{T}\ \mathbf{S})^T = \mathbf{S}^T\ \mathbf{T}^T$.

Q.2 Given any skew tensot $\mathbf{\Omega}$, show that there is a unique vector $\boldsymbol{\omega}$ – called the axial vector of $\mathbf{\Omega}$ – such that

$$\mathbf{\Omega u} = \boldsymbol{\omega} \times \mathbf{u}$$

for all vectors $\mathbf{u}$.

Q.3 For the tensor:

$$\mathbf{T} = \begin{matrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 3 \end{matrix}$$

Find it's scalar invariants. Also obtain the eigenvalues by solving $\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0$.

Q.4 Given the scalar field $\varphi(\mathbf{x}) = \mathbf{x}\cdot\mathbf{a} + \mathbf{x}\cdot\mathbf{x}$, where $\mathbf{a}$ is constant vector. Find the gradient by first finding the *directional* derivative. Also determine the components of the gradient.

Section B

Q.5 Uniaxial compression is defined by the displacement field

$$\mathbf{u} = -\varepsilon X_1 \mathbf{e}_1.$$

Determine the normal strain for the material fibers oriented at 30° to the $\mathbf{e}_1$–direction in the $(\mathbf{e}_1, \mathbf{e}_2)$–plane.

Q.6 Prove Nanson's Formula i.e. show that $\mathbf{n}da = J\mathbf{F}^{-\mathbf{T}}\mathbf{N}dA$.

Q.7 Show that $\dot{\mathbf{J}} = Jdiv\mathbf{v}$. Using this result and the localization theorem derive the local form of the conservation of mass.

Q.8 Present the global form of the balance of linear and angular momemtun. Clearly explain each term. Next, derive their local forms.

x-x-x