Exam. Code: 0927
Sub. Code: 6571

2074

B.E. (Electronics and Communication Engineering)
Third Semester
MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.

x-x-x

1. (a) Define a basis of vector space. Find a standard basis vector that can be added to the set $S = \{(1, 0, 3), (2, 1, 4)\}$ to produce the basis of R^3.

(b) Define: (i) homomorphism of a vector space, (ii) range of linear transformation.

(c) Define eigenvalue problem of matrix. The eigenvalues of a matrix A are 1, 2, 3, then find the eigenvalues of $A^2 + A^{-1}$.

(d) Define bounded function of a complex variable. Prove that $w = \cos z$ is not a bounded function.

(e) Explain the difference between zeros and singularities of a function. Find the same and discuss the nature of singularities of $f(z) = \frac{1-\cos z}{z}$.

SECTION-A

2. (a) Solve the homogeneous system $AX = 0$, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. Find the rank of A and nullity(A).

(b) FInd conditions on p, q, r so that $r = (p, q, r)$ in R^3 belong to $S = \text{span}(u, v, w)$, where $u = (1, 2, 0)$, $v = (-1, 1, 2)$, $w = (3, 0, -4)$.

(c) Prove that all lower triangular matrices of order 2 over R is a vector space with standard matrix addition and scalar multiplication. (3+3+4)

3. (a) State Cayley-Hamilton theorem. Use it to find A^{-1} for $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

(b) Find a matrix P which diagonalizes the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ and hence determine $P^{-1}AP$.

4. (a) Find $T(x, y)$, where $T: R^2 \to R^3$ is defined as $T(2, -5) = (-1, 2, 3,)$ and $T(3, 4) = (0, 1, 5)$.

(b) Verify rank-nullity theorem for linear transformation defined as

$$T(x, y, z) = (2x, 4x - y, 2x + 3y - z).$$

(c) Let T be a linear operator on R^3 defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$. Find the matrix of T relative to the basis $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$.

SECTION-B

5. (a) Write down any three similarities and differences between $\cos x$ and $\cos z$.

(b) Examine whether the function $f(z) = \begin{cases} \frac{Re(z^2)}{|z|^2}, & \text{if } z \neq 0 \\ 0, & \text{if } z = 0 \end{cases}$ is continuous at $z = 0$.

(c) Define an analytic function and prove that an analytic function of constant modulus is constant.

(3 + 3 + 4)

6. (a) Find all the possible Taylor and Laurent series of $f(z) = \frac{1}{z^2 - 3z + 2}$ in

(i) $1 < |z| < 2$, (ii) $0 < |z - 1| < 1$, (iii) $|z| > 2$.

(b) Determine and classify the zeros and singular points of the following functions:

(i) $f(z) = \frac{\sin z}{(z-\pi)^2}$, (ii) $f(z) = \tan\left(\frac{1}{z}\right)$, (iii) $f(z) = (z+i)^2 e^{\frac{1}{z+i}}$.

7. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$ using contour integration.

(b) Find the linear fractional transformation that maps $(-2, 0, 2)$ onto $\left(\infty, \frac{1}{4}, \frac{3}{8}\right)$.

Under this map, what is the image of x-axis.

x-x-x