2074

B.E. (Computer Science and Engineering)
Third Semester
CS-303: Discrete Structures

Time allowed: 3 Hours

Max. Marks: 50

*NOTE: Attempt **five** questions in all, including Question No. I which is compulsory and selecting two questions from each Section.*

x-x-x

1. Briefly explain the following using a suitable exanple:

   (a) Counting principles

   (b) Order and degree of recurrence relation

   (c) Existential Quantifier

   (d) Poset

   (e) Group.

   (5x2=10)

Section-A

2. (a) Show that $X \subseteq Y$ implies $X \times Z \subseteq Y \times Z$.
      (2)
   (b) Show that the function $f(x) = ax + b$ from R to R is invertible, where a and b are constants, with a not equal to 0, and find the inverse of f .
      (2)
   (c) Find the symmetric difference of the set of computer science majors at a school and the set of mathematics majors at a school.
      (1)
   (d) Prove that for all non-negative real numbers x,y,z if $x^2 + y^2 = z^2$ then $x + y >= z$ using proof by contradiction.
      (5)

3. Check the validity of the following arguments using rules of inference of propositional logic:

   (i) If you send me an e-mail message, then I will finish writing the program. If you do not send me an e-mail message, then I will go to sleep early. If I go to sleep early, then I will wake up feeling refreshed. If I do not finish writing the program, then I will wake up feeling refreshed.

   (ii) Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution.

   (5+5=10)

4. (i) Let R be the relation on the set of real numbers such that xRy if and only if x and y are real numbers that differ by less than 1, that is $|x - y| < 1$. Show that R is not an equivalence relation.

   (5)

(*ii*) Determine whether the posets represented by each of the Hasse diagrams (a), (b), (c), (d) in the figure given below have a greatest element and a least element. Justify your answer.

(5)

(a) (b) (c) (d)
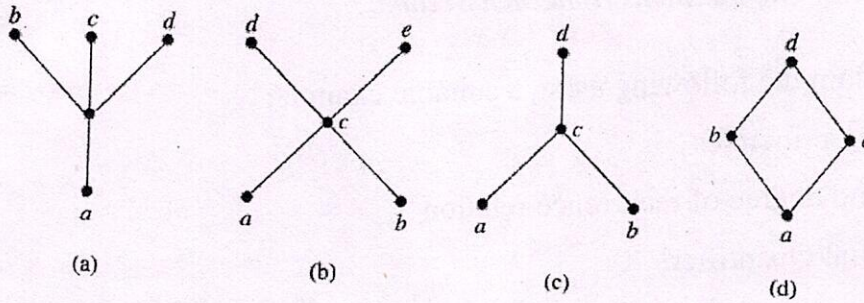
Section-B

5. (a) Solve the recurrence relation: $x_n - 5x_{n-1} + 6x_{n-2} = 2^n + n$, $n >= 2$, $x_0 = 1$, $x_1 = 1$ by the method of generating functions.

(5)

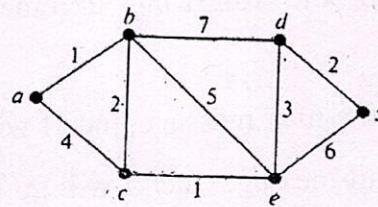(b) Find the number of different outcomes when three dice are rolled.

(5)

6. (a) Contrast between Euler graph and Hamiltonian circuit with suitable examples. (5)

(b) Discuss Dijkstra's algorithm to find the shortest path for the given graph. (5)

7. (a) Suppose that there are eight runners in a race. The winner receives a gold medal, the secondplace finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

(5)

(b) Prove that the fourth roots of unity 1, -1, i, -i form an abelian multiplicative group?

(5)

x-x-x