Exam.Code:0933
Sub. Code: 6660

2074

B.E. (Electrical and Electronics Engineering)

Third Semester

BS-EE-305: MATH-III

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks. Use of a simple calculator is allowed.*

x-x-x

1. (a) Define linear combination, linearly dependent and independent vectors. Find α if the vectors $\begin{bmatrix}1\\-1\\3\end{bmatrix}, \begin{bmatrix}1\\2\\-3\end{bmatrix}, \begin{bmatrix}\alpha\\0\\1\end{bmatrix}$ are linearly independent.

(b) Define linear span and basis of a vector space with suitable examples.

(c) Define linear transformation. Examine whether a map is defined by $T(x, y) = (e^x, e^y)$ is linear or not.

(d) Derive C-R equations in polar coordinates.

(e) Define conformal and isogonal mappings with suitable examples. Discuss the mapping $w = kz$.

(5×2)

SECTION-A

2. (a) Determine when the augmented matrix represents a consistent linear system:
$$x + 2y = a;\ 2x + y + 5z = b;\ x - y + z = c.$$

(b) Determine whether the vector $v = \begin{bmatrix}-5\\11\\-7\end{bmatrix}$ is a linear combination of the vectors
$$v_1 = \begin{bmatrix}1\\-2\\2\end{bmatrix}, v_2 = \begin{bmatrix}0\\5\\5\end{bmatrix}, v_3 = \begin{bmatrix}2\\0\\8\end{bmatrix}.$$ (3 + 3 + 4)

(c) Define subspace of a vector space. Let V be a vector space of the R^3. Examine, whether the following are subspaces of V or not?

(i) $W = \{(x, y, z): 3x + y - z = 0, x, y, z \in R\}$,

(ii) $W = \{(x, y, z): xy = 0, x, y, z \in R\}$.

3. (a) State Cayley-Hamilton theorem. Find A^{-1}, it it exists where $A = \begin{bmatrix}1 & 1 & 2\\1 & 3 & -1\\-2 & -1 & 1\end{bmatrix}$.

(b) Examine whether the matrix: $A = \begin{bmatrix}3 & 1 & -1\\-2 & 1 & 2\\0 & 1 & 2\end{bmatrix}$ is diagonalozable or not? If yes, then, find P such that $P^{-1}AP$ is a diagonalizable.

(2×5)

4. (a) Let T: $R^3 \to R^3$ defined by $T(x, y, z) = (x + y, z, x - y)$. Find range, kernel, nullity and rank of T.

(b) Find a linear transformation T: $R^3 \to R^2$ such that $T(1, 1, 0) = (1, 0)$ and $T(1, -1, 0) = (1, 1)$. Also find $T(10, 50, 7)$.

(c) Consider the two bases of R^2: $S = \{(1, 2), (3, 5)\}$ and $S^1 = \{(1, -1), (1, -2)\}$. Find the change of basis matrix from S to S^1 and vice versa. (3+3+4)

SECTION-B

5. (a) Find all the values which satisfy $\sin\left(\frac{i}{z}\right) = i$.

(b) Define analytic function. Check whether $f(z) = \log z$ is analytic.

(c) Define harmonic function. Prove that $u = \sin x \cosh y$ is harmonic. Hence, find its harmonic conjugate.

(3 + 3 + 4)

6. (a) State Taylor and Laurent's expansion. Explain the differences between them. Find the possible expansion for $f(z) = \frac{z^2-4}{(z+1)\ (z+4)}$, which are valid for the regions: (i) $1 < |z| < 4$, (ii) $|z| > 4$.

(b) Write the principal part of the function $f(z) = z \exp\left(\frac{1}{z}\right)$ at its isolated singular point and determine whether that point is a pole, removable singularity or an essential singular point.

(4 + 2 + 4)

(c) Compute the residues at all the isolated singular points in the finite complex plane of the function f(z): (i) $f(z) = \frac{\sin z}{z^2+1}$, (ii) $f(z) = \frac{\cot z}{z}$.

7. (a) Evaluate the real definite integral using contour integration:

$$\int_0^{2\pi} \frac{\cos 3\theta}{5-3\cos\theta}\, d\theta.$$

(4 + 4 + 2)

(b) Find the bi-linear transformation which maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ respectively.

(c) Find the bi-linear map whose fixed points are -1 and 1.

x-x-x