Exam.Code:0921
Sub. Code: 6471

2074

B. E. (Information Technology)

Third Semester

ASM-301: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.*

x-x-x

Attempt the following:-

1 (a) Under what conditions on a, the vectors $(1+a, 1-a)$ and $(a-1, 1+a)$ are linearly independent. (1.5)

(b) Is the map $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ defined by

$$T(x, y) = (2x - y, x + y, -x - 3y),$$

linear? Prove or disprove? (1.5)

(c) Show that λ^2 is an eigen value of A^2. (1.5)

(d) The painted light bulbs produced by a factory are 50 percent red, 30 percent green, 20 percent blue. In a sample of five bulbs, find the probability of getting 2 red, 1 green, and 2 blue. (2)

(e) Find the probability of selecting a black card or a six from a deck of 52 cards. (1.5)

(f) Let A and B be events with $P(A) = 0.6$, $P(B) = 0.3$, and $P(A \cap B) = 0.2$. Find $P(A^c|B^c)$ and $P(B^c|A^c)$. (2)

Section-A

2 (a) Let V be a vector space of polynomials of degree ≤ 3 over reals. Let $T : V \longrightarrow V$ defined by

$$F(a + bx + cx^2 + dx^3) = a + b(x+1) + c(x+1)^2 + d(x+1)^3.$$

Find the matrix of T relative to the basis $\{1,\ 1+x,\ 1+x^2,\ 1+x^3\}$. (5)

(b) State and prove Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{pmatrix}$ (5)

P.T.O.

3 (a) Let $B_1 = \{(1,2,0), (1,3,2), (0,1,3)\}$ and $B_2 = \{(1,2,1), (0,1,2), (1,4,6)\}$. Find the change of basis matrix from B_1 to B_2. (5)

(b) Solve the linear system of equations

$$-3x + 2y + 4z = 12,\ x - 2z = -4,\ 2x - 3y + 4z = -3,$$

using row reduction method. (5)

4 (a) Check, whether the set $\{4x - x^2,\ 5 + x^3,\ 3x + 5,\ 2x^3 - 3x^2\}$ is a basis of vector space $\mathbb{P}_3$ of polynomials of degree ≤ 3 over reals or not? (5)

(b) Determine whether the transformation $T : \mathbb{P}_2 \longrightarrow \mathbb{R}^3$ defined by

$$T(a + bx + cx^2) = (a - c, 2b, a + c).$$

has an inverse and, if so, determine T^{-1}. (5)

Section - B

5 (a) A man fires a target six times and hit it two times. List the different ways that this can happen and how many different ways are there? (3)

(b) Let X_1, X_2, X_3 be independent random variables that are identically distributed with mean $\mu = 100$ and standard deviation $\sigma = 4$. Let $Y = (X_1 + X_2 + X_3)/3$. Find mean μ_Y of Y and standard deviation σ_Y of Y. (4)

(c) The batting average of a baseball player is 300. He comes to bat four times. Find the probability that he will get: (a) exactly two hits, (b) at least one hit. (3)

6 (a) Let X be a random variable with $M_X(t) = e^{4(e^t - 1)}$. Find $E(X)$ and $V(X)$. (3)

(b) The joint probability density function of X and Y is given by

$$f(x,y) = e^{-y},\ \ 0 < x < y, 0 < y < \infty.$$

Find the probability density function of $Z = X/Y$. (4)

(c) Prove that $\text{var}(aX + b) = a^2\ \text{var}(X)$. (3)

7 (a) If 2 gallon can of paint covers on an average 800 sft. with standard deviation 80 sft., what is the probability that the mean area covered by the sample of 60 of these 2 gallon cans will be anywhere from 750 to 820 sft.? (5)

(b) Let X and Y have joint probability density function

	Y = -1	Y = 0	Y = 1
X = 0	Y = b	Y = 2b	Y = b
X = 1	Y = 3b	Y = 2b	Y = b
X = 2	Y = 2b	Y = b	Y = 2b

Find marginal distributions of X and Y. Also find conditional distributions of X and Y. (5)

x-x-x