Exam.Code:0905
Sub. Code: 6210

2074

**B.E., First Semester**
**MATHS-101: Calculus**
**(Common to all Streams)**

**Time allowed: 3 Hours**
**Max. Marks: 50**

***NOTE:*** ***Attempt <u>five</u> questions in all, including Question No. I which is compulsory and selecting two questions from each Part. Use of non-programmable calculator is allowed.***

*x-x-x*

1. (a) Find the domain, range, level curves of the function $f(x, y) = 1 - |x| - |y|$. Also draw its graph.

   (b) Find $\lim\limits_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$, if it exists.

   (c) Define curvature and torsion for a smooth curve in space.

   (d) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, given that $f(x, y, z) = 0$ defines $z$ as an implicit function of two independent variables $x$ and $y$.

   (e) Find the area of the circle $r = a$ using polar coordinates. $(5 \times 2 = 10)$

## PART A

2. (a) Check the absolute and conditional convergence of the alternating series: (3)

$$\sum_{1}^{\infty}(-1)^{(n+1)}\frac{n!}{2^n}$$

   (b) Check the convergence of the sequence: (3)

$$a_n = \frac{(2n+3)!}{(n+1)!}$$

   (c) Check the convergence of the series and find the interval of convergence of the series: (4)

$$\sum_{n=1}^{\infty}\frac{(3x-4)^n}{\sqrt[3]{n}}$$

3. (a) Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$, and the x-axis. (3)

   (b) Find the absolute maxima and minima of the function: $f(x, y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0, y = 0, y + 2x = 2$ in the first quadrant. (4)

   (c) Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis, and on the right by the line $x = 2$ about y-axis. (3)

P.T.O.

4. (a) If $w = f(u, v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$ and if $u = (x^2 - y^2)/2$ and $v = xy$, then find the value of $w_{xx} + w_{yy}$. (5)

   (b) Find the maximum value that $f(x, y, z) = x^2 + 2y - z^2$ can have on the line of intersection of planes 2x-y=0 and y+z=0. (5)

## PART B

5. (a) Find the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cylinder $x^2 + y^2 = 1$. (5)

   (b) Find the volume of the solid right cylinder $D$ whose base is the region in the xy-plane that lies inside the cardioid $r = 1 + \cos(\theta)$ and outside the circle $r = 1$ and whose top lies in the plane $z = 3$. (5)

6. (a) Show that the differential form in the integral is exact. Then evaluate the integral. (5)

$$\int_{(1,1,2)}^{(3,5,0)} (yz\ dx + xz\ dy + xy\ dz)$$

   (b) Find the outward flux of the field (5)

$$\mathbf{F} = (3xy - x/(1 + y^2))\ \hat{\imath} + (e^x + \tan^{-1} y)x)\ \hat{\jmath}$$

   across the cardioid $r = a(1 + \cos(\theta)), a > 0$.

7. (a) Find curvature ($\kappa$) and torsion ($\tau$) for the following curve: (6)

$$\mathbf{r(t)} = \ln(sect)\ \hat{\imath} + t\ \hat{\jmath}, -\pi/2 < t < \pi/2$$

   (b) Show that the vector-valued function (4)

$$\mathbf{r(t)} = (2\,\hat{\imath}+2\,\hat{\jmath}+\hat{k}) + \cos(t)\left(\frac{1}{\sqrt{2}}\,\hat{\imath} - \frac{1}{\sqrt{2}}\,\hat{\jmath}\right) + \sin(t)\left(\frac{1}{\sqrt{3}}\,\hat{\imath} + \frac{1}{\sqrt{3}}\,\hat{\jmath} + \frac{1}{\sqrt{3}}\,\hat{k}\right)$$

   describes the motion of a particle moving in the circle of radius 1 and centered at the point (2,2,1) and lying in the plane $x + y - 2z = 2$.

x-x-x