Exam.Code:0905
Sub. Code: 6193

2074

B.E., First Semester

ASM-101: Calculus

(Common to all streams)

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt <u>five</u> questions in all, including Question No. I which is compulsory and selecting two questions from each Section.

x-x-x

1. (a) Test the convergence or divergence for the sequence $\{\frac{n!}{n^n}\}$ and find

$$\lim_{n\to+\infty} \frac{n!}{n^n}.$$

(b) What is a potential function? Show by example how to find a potential function for a conservative field. How can you tell when a field is conservative?

(c) Find the linearizations of the function $f(x, y, z) = \sqrt{2}\cos x \sin(y+z)$ at the point $(\frac{\pi}{4}, \frac{\pi}{4}, 0)$

(d) Let D be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 4$ and on the sides by the cylinder $x^2 + y^2 = 1$. Set up the triple integrals in spherical coordinates for volume and find the limits of D using the $d\rho\, d\phi\, d\theta$ orders of integration.

(e) Find curvature and torsion for

$$\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k}, \quad a, b \geq 0, a^2 + b^2 \neq 0.$$

[02, 02, 02, 02, 02]

Section A

2. (a) Test the convergence or divergence of p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}, \quad (p \text{ is a real constant})$$

for $p > 1$, and for $p \leq 1$.

(b) State the Integral Test and use it to test the convergence or divergence of the series:

$$\sum_{n=3}^{\infty} \frac{(1/n)}{(ln\ n)\sqrt{ln^2\ n - 1}}.$$

[05, 05]

P.T.O.

3. (a) Find the minimum volume for a region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and a plane tangent to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at a point in the first octant.

   (b) The surface $f(x, y, z) = x^2 + y^2 - 2 = 0$ and $g(x, y, z) = x + z - 4 = 0$ meet in an ellipse E. Find the parametric equations for the line tangent to E at the point $P(1, 1, 3)$.

[05, 05]

4. (a) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.

   (b) Find an equation for the plane tangent to the level surface $x^2 - y - 5z = 0$ at the point $P_0(2, -1, 1)$. Also, find parametric equations for the line that is normal to the surface at $P_0(2, -1, 1)$.

   (c) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

[04, 03, 03]

Section B

5. (a) Let the region D in xyz-space defined by the inequalities $1 \le x \le 2$, $0 \le xy \le 2$, $0 \le z \le 1$. Evaluate

$$\iiint_D (x^2 y + 3xyz)\, dx\, dy\, dz$$

   by applying the transformation $u = x$, $v = xy$, $w = 3z$ and integrating over an appropriate region G in uvw-space.

   (b) Find the area of the region common to the interiors of the cardioids $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

   (c) Find the volume of the "ice cream cone" D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$.

[04, 03, 03]

6. (a) Show that the curvature of a smooth curve $\vec{r}(t) = f(t)\hat{i} + g(t)\hat{j}$ defined by twice-differentiable functions $x = f(t)$ and $y = g(t)$ is given by the formula

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}.$$

(b) Find $\vec{T}$, $\vec{N}$, $\vec{B}$, κ and τ for the space curve

$$\vec{r}(t) = (\cos t + t\sin t)\hat{i} + (\sin t - t\cos t)\hat{j} + 3\hat{k}.$$

[05, 05]

7. (a) Verify both forms of Green's Theorem for the field

$$\vec{F}(x, y) = (x - y)\hat{i} + x\hat{j}$$

and the region R bounded by the unit circle

$$C : \vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j}, \quad 0 \le t \le 2\pi.$$

(b) Find the flux of $\vec{F} = yz\hat{j} + z^2\hat{k}$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1$, $z \ge 0$, by the planes $x = 0$ and $x = 1$.

[06, 04]

x-x-x