Exam.Code: 0905 Sub. Code: 6193

2074 B.E., First Semester ASM-101: Calculus (Common to all streams)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt <u>five</u> questions in all, including Question No. I which is compulsory and selecting two questions from each Section.

x-x-x

1. (a) Test the convergence or divergence for the sequence $\{\frac{n!}{n^n}\}$ and find

$$\lim_{n\to+\infty}\frac{n!}{n^n}.$$

- (b) What is a potential function? Show by example how to find a potential function for a conservative field. How can you tell when a field is conservative?
- (c) Find the linearizations of the function $f(x, y, z) = \sqrt{2} \cos x \sin(y+z)$ at the point $(\frac{\pi}{4}, \frac{\pi}{4}, 0)$
- (d) Let D be the region bounded below by the plane z=0, above by the sphere $x^2+y^2+z^2=4$ and on the sides by the cylinder $x^2+y^2=1$. Set up the triple integrals in spherical coordinates for volume and find the limits of D using the $d\rho d\phi d\theta$ orders of integration.
- (e) Find curvature and torsion for

$$\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k}, \quad a, b \ge 0, a^2 + b^2 \ne 0.$$

[02, 02, 02, 02, 02]

Section A

2. (a) Test the convergence or divergence of p-series

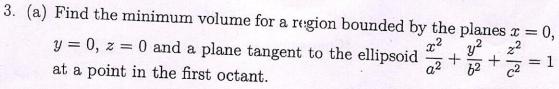
$$\sum_{n=1}^{\infty} \frac{1}{n^p}, \quad (p \text{ is } \mathbf{a} \text{ real constant})$$

for p > 1, and for $p \le 1$.

(b) State the Integral Test and use it to test the convergence or divergence of the series:

$$\sum_{n=3}^{\infty} \frac{(1/n)}{(\ln n)\sqrt{\ln^2 n - 1}}.$$

[05, 05]



(b) The surface $f(x, y, z) = x^2 + y^2 - 2 = 0$ and g(x, y, z) = x + z - 4 = 0 meet in an ellipse E. Find the parametric equations for the line tangent to E at the point P(1, 1, 3).

[05, 05]

- 4. (a) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.
 - (b) Find an equation for the plane tangent to the level surface $x^2 y 5z = 0$ at the point $P_0(2, -1, 1)$. Also, find parametric equations for the line that is normal to the surface at $P_0(2, -1, 1)$.
 - (c) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines y = 1, x = 4 about the line y = 1.

[04, 03, 03]

Section B

5. (a) Let the region D in xyz-space defined by the inequalities $1 \le x \le 2$, $0 \le xy \le 2$, $0 \le z \le 1$. Evaluate

$$\iiint_D (x^2y + 3xyz) \, dx \, dy \, dz$$

by applying the transformation $u=x,\ v=xy,\ w=3z$ and integrating over an appropriate region G in uvw-space.

- (b) Find the area of the region common to the interiors of the cardioids $r=1+\cos\theta$ and $r=1-\cos\theta$.
- (c) Find the volume of the "ice cream cone" D cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$.

[04, 03, 03]

6. (a) Show that the curvature of a smooth curve $\vec{r}(t) = f(t)\hat{i} + g(t)\hat{j}$ defined by twice-differentiable functions x = f(t) and y = g(t) is given by the formula

 $\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}.$

(b) Find \vec{T} , \vec{N} , \vec{B} , κ and τ for the space curve

$$\vec{r}(t) = (\cos t + t \sin t)\hat{i} + (\sin t - t \cos t)\hat{j} + 3\hat{k}.$$

[05, 05]

7. (a) Verify both forms of Green's Theorem for the field

$$\vec{F}(x,y) = (\mathbf{x} - y)\hat{i} + x\hat{j}$$

and the region R bounded by the unit circle

$$C: \vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j}, \quad 0 \le t \le 2\pi.$$

(b) Find the flux of $\vec{F} = yz\hat{j} + z^2\hat{k}$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1$, $z \ge 0$, by the planes x = 0 and x = 1.

[06, 04]