Exam.Code:0918
Sub. Code: 6365

2014

B.E. (Computer Science and Engineering)
Sixth Semester
CS-602: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section. Use of a statistical table and simple calculator is allowed. Each question carries equal marks.

x-x-x

1. (a) Define echelon form of a matrix. Why is the echelon form of a matrix important?

Is this matrix: $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ in echelon form? Justify.

(b) Define linear combination of vectors along with its importance. Can the polynomial $3x^2 - 5x + 7$ be expressed as a linear combination of the polynomials $2x^2 + 7x - 3$ and $x^2 + 3x - 5$.

(c) Prove that a mapping: $T: R^3 \to R^2$ defined by $T(x, y, z) = (x, y)$ is a linear transformation.

(d) Why are different kinds of probability distribution appear in the study of probability? Why is normal distribution more popular than other distributions?

(e) Define joint distributions along with its properties.

SECTION-A

2. (a) Reduce the matrix $A = \begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 4 & -1 & 5 \end{bmatrix}$ to column echelon form and find its rank.

(b) Solve the homogeneous system: $AX = 0$, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. Also find out rank of A and nullity(A).

(c) Define subspace of a vector space. Let V be a subspace of R^3. Examine whether the following is a subspace of V or not?

$$W = \{(x, y, z): 3x + y - z = 0, x, y, z \in R\}.$$ (3 + 3 + 4)

3. (a) Find the condition on u, v, w so that (u, v, w) belongs to the vector space generated by (1, 2, 3) and (−1, 2, 4).

(b) Prove that the set $\{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$ forms a basis of R^3.

(c) Find a linear transformation: $R^3 \to R^2$ such that $T(1, 1, 1) = (1, 0)$ and $T(1, 1, 2) = (1, -1)$. Also verify your answer.

(3+3+4)

P.T.O.

4. (a) Find the eigenvalues and the corresponding eigenvectors of the matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

(b) The vectors $u_1 = (1, 2, 0), u_2 = (1, 3, 2)$, $u_3 = (0, 1, 3)$ form a basis S of R^3. Find: (i) The change of basis matrix from the usual basis $E = \{e_1, e_2, e_3\}$. (ii) The change of basis matrix Q from S back to E.

SECTION-B

5. (a) State and prove Baye's theorem on conditional probability.

(b) A random variable X has the following probability distribution

x	: 0	1	2	3	4
p(x)	: c	2c	2c	c^2	$5\,c^2$

Find the value of c. Evaluate $P(X < 3), P(0 < X < 4)$. Find the distribution of X. Find the mean and variance.

6. (a) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals (i) exactly 3, (ii) more than 2 individuals, (iii) none, (iv) more than one individual will suffer a bad reaction.

(b) Write a note on normal distribution along with its applications.

7. (a) Define joint distribution along with its properties. Explain transformation of variables in joint distribution.

(b) The pdf of the random variable (X, Y) is given by

$$f(x, y) = \begin{cases} \frac{1}{4} e^{-\frac{(x+y)}{2}}, x > 0, y > 0, \\ 0, \text{elsewhere.} \end{cases}$$ find the distribution of $\frac{X-Y}{4}$.

x-x-x