2014

B.E. (Information Technology) Fourth Semester

ASM-401: Discrete Structures

Time allowed: 3 Hours   Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. 1 (Section-A) which is compulsory and selecting two questions each from Section B-C.*

x-x-x

Section - A

1. Answer the following:

   a) Let $R = \{(1,2), (2,2), (3,4), (4,1)\}$ be any relation. Check whether R is symmetric, antisymmetric or transitive. Also, find symmetric, antisymmetric and transitive closure of R.

   b) Find the coefficient of x^5 in $(1 - 2x)^{-7}$.

   c) Define Group and Subgroup.

   d) Define Linear ordered set. What do you mean by Lexicographer order? Give examples.

   e) Prove that G is abelian group iff $(ab)^{-1} = a^{-1}b^{-1}$ $\forall a, b \in G$.

   $(5 \times 2 = 10)$

Section - B

2. a) Let $\sim$ be an equivalence relation on A.

   (i) Prove that if $a \sim b$ then $[a] = [b]$: for any x,

   $$x \in [a] \iff x \in [b]$$

   (ii) Prove that if $a \nsim b$, then $[a] \cap [b] = \emptyset$.

   b) Let $S = \{a, b, c\}$ and $P(S)$ be the power set of S. Prove that $(P(S), \subseteq)$ is a POSET with partial order $\subseteq$ and draw the Hasse diagram of it.

   c) Define distributive lattice. Prove that if a and b are elements in a bounded distributive lattice and if a has a complement a', then

   $$a \vee (a' \wedge b) = a \vee b$$
   $$a \wedge (a' \vee b) = a \wedge b.$$

   $(04 + 03 + 03)$

3. a) Prove that if 101 integers are selected from the set $S = \{1, 2, 3, \ldots, 200\}$, then there are two integers such that one divides the other.

   b) Suppose 63 seats in a class room are arranged with 7 rows of 9 seats each. Suppose 40 students are seated randomly. Show that each row has at least 6 students and some column has at most 4 students.

c) Let S be a set of six positive integers whose maximum is at most 14. Show that the sums of the elements in all the nonempty subsets of S cannot be all distinct.

(04+03+03)

4. a) Check the validity of the argument:

If there is a chance of rain or her red headband is missing, then Lois will not mow her lawn. Whenever the temperature is over 80°F, there is no chance for rain. Today the temperature is 85"F and Lois is wearing her red headband. Therefore Lois will mow her lawn.

b) Let $p(x), q(x)$ be open statements in the variable x, with a given universe. Prove that

$$\forall x\ p(x) \vee \forall x\ q(x) \Rightarrow \forall x\ [p(x) \vee q(x)]$$

Also find the counterexample for the converse.

(05 + 05)

Section - C

5. a) Define the following with suitable examples:

   i. Bipartiate Graph
   ii. Chromatic number of graph
   iii. Euler circuit and Hamiltonian circuit
   iv. Isomorphic graphs

b) Six married couples are to be seated at a circular table. In how many ways they can arrange themselves so that no wife sits next to her husband?

(05 + 05)

6. a) Solve the following recurrence relation using method of generating functions:

$$a_{n+2} - 5a_{n+1} + 6a_n = 2, \ n \geq 0, \quad a_0 = 3, \quad a_1 = 7.$$

b) For $n \geq 0$, let $S = \{1, 2, 3, \ldots, n\}$ and let a_n denote the number of subsets of S that contain no consecutive integers. Find and solve recurrence relation for a_n.

(05 + 05)

7. a) Define Group. What is the difference between abelian group and cyclic group. Prove that every cyclic group is abelian where converse is not true. Give an example of a smallest non-cyclic abelian group.

b) If G be a group with H finite and $H \subseteq G$, then H is a subgroup of G if and only if H is closed under the binary operation G.

(05 + 05)