Exam. Code: 0927
Sub. Code: 33641

2124

B.E. (Electronics and Communication Engineering)
Third Semester
MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.

x-x-x

1. (a) Explain the relation between basis and dimension of a vector space. Why does every vector space have a basis? Explain.

   (b) Define similar and diagonalizable matrices. Write different properties of similar matrices. What is the purpose of diagonalizing a matrix?

   (c) State rank-nullity theorem. Explain its significance.

   (d) Prove that $w = \cos z$ is an unbounded function. List out the differences and similarities between $\cos x$ and $\cos z$.

   (e) List the different types of singularities of a function $w = f(z)$. Identify the type of singularity for $f(z) = \sin\left(\frac{1}{z}\right)$ about $z = 0$.

SECTION-A

2. (a) Define a homogeneous system of linear equations and explain its significance in linear algebra. Describe under what conditions a homogeneous system has a non-trivial solutions. $(3 + 4 + 3)$

   (b) Solve the homogeneous system of equations $AX = 0$, where

   $$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 1 & 4 \\ 3 & 2 & -6 & 1 \end{bmatrix}$$. Also find rank (A) and nullity(A).

   (c) Express the matrix $A = \begin{bmatrix} 2 & 0 \\ 4 & -5 \end{bmatrix}$ as a linear combination of the matrices:

   $A = \begin{bmatrix} 0 & -3 \\ 4 & -5 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$.

3. (a) Define a vector space. Check whether the set of all pairs of real numbers of the form $(1, x)$ with the operations $(1, x_1) + (1, x_2) = (1, x_1 + x_2)$ and $k(1, x) = (1, k x)$ is a vector space.

   (b) Let $S = (u, v, w)$, where $u = (1, -3, -2)$, $v = (-3, 1, 3)$ and $w = (-2, -10, -2)$. Verify whether S forms a basis or not.

P.T.O.

4. (a) Test the matrix: $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \in M_{2\times2}(R)$ for diagonalizablity. If diagonalizable, then diagonalize it.

(b) Find the rank and nullity of T, where $T: V \rightarrow W$ be a linear transformation defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. (3 + 3 + 4)

(c) Find the matrix of the linear transformation $T: R^2 \rightarrow R^3$ defined by $T(x, y) = (x = y, x, 3x - y)$ with respect to the basis $B_1 = \{(1,1), (3,1)\}$ and $B_2 = \{(1,1,1), (1,1,1), (1,0,0)\}$.

SECTION-B

5. (a) Describe the concept of continuity and differentiability for complex functions. How does continuity of complex functions compare with that of real functions?

(b) Prove that $|sin\ z\ |^2 = sin^2x + sinh^2y$. Find the values of z for which $\sin z = 0$.

(c) Give an example of function, which is continuous, differentiable but not analytic with proper justification. (3 + 3 + 4)

6. (a) If f(z) is analytic in a region and if $|f\ (z)|$ is constant there, then prove that f(z) is a constant function.

(b) Obtain Taylor's and Laurent's series expansion for $f(z) = \frac{(z-2)\ (z+2)}{(z+1)\ (z+4)}$, which are valid: (i) when $|z| < 1$, (ii) $1 < |z| < 4$, (iii) $|z| > 4$.

7. (a) Evaluate $\int_0^{\pi} \frac{1}{2+sin\theta}\ d\theta$ by residue theorem.

(b) Find all the bi-linear transformation whose fixed points are -1 and 1.

(c) Discuss the mapping $w = \frac{1}{z}$. Find the image of the strip $2 < x < 3$.

(4 + 3 + 3)

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