

2124

B.E. (Computer Science and Engineering)

Third Semester

CS-303: Discrete Structures

Time allowed: 3 Hours

Max. Marks: 50

**NOTE:** Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.

 $x-x-x$ 

1. Briefly explain the following with example:

- (a) Reflexive and Irreflexive relations
- (b) K-Regular graph
- (c) Monoid
- (d) Recurrence relation
- (e) Quantifiers

(5x2=10)

## Section-A

2. (a) Consider the universal set  $U=\{1,2,3,4,\dots,10\}$  and the subsets  $A=\{1,7,8\}$ ,  $B=\{1,6,9,10\}$ ,  $C=\{1,9,10\}$

I. List the non-empty minsets generated by A, B and C. Do the minsets form a partition of U?

II. How many elements of U can be generated by A, B and C?

III. Compare the number obtained in II. with  $n(P(U))$ .

(4)

(b) If R be a relation in the set of integers Z defined by  $R = \{(x, y): x \in Z, y \in Z, (x-y) \text{ is divisible by } 6\}$ . Then prove that R is an equivalence relation.

(4)

(c) Is the Implication and its inverse logically equivalent? Justify your answer.

(2)

3. (a) Show that the mapping  $f: R \rightarrow R$  be defined by  $f(x) = ax + b$ , where  $a, b, x \in R$ ,  $a \neq 0$  is invertible. Find its inverse.

(4)

(b) Determine the negation of the following statements

- I.  $\forall x \forall y \forall z, p(x, y, z)$
- II.  $\forall x \exists y, p(x, y)$
- III.  $\forall x \forall y (p(x) \wedge q(y))$

(3)

(c) Consider the function  $f: N \rightarrow N$ , where N is the set of natural numbers including zero defined by  $f(n) = n^2 + 2$ . Check whether the function f is (i) one-one (ii) onto.

(3)

4. (a) Let  $D_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$  whose all the elements are divisors of 100. Let the relation  $<$  be the relation | (divides) be a partial ordering on  $D_{100}$ .

Contd.....P/2



(2)

- I. Draw the Hasse diagram of the given poset.
- II. Determine the glb of  $\{10, 20\}$  and  $\{5, 10, 20, 25\}$
- III. Determine the lub of  $\{10, 20\}$  and  $\{5, 10, 20, 25\}$

(5)

(b) Prove the validity of following arguments without using truth tables.

- I.  $p \vee q, \neg p \vdash q$
- II.  $p, p \rightarrow q, q \rightarrow r \vdash r$
- III.  $p \rightarrow (q \vee r), (s \wedge t) \rightarrow q, (q \vee r) \rightarrow (s \wedge t) \vdash p \rightarrow q$

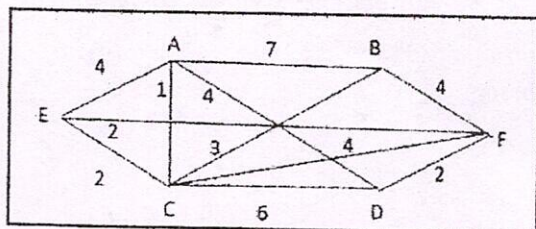
(3)

(c) Let  $R$  be a binary relation on  $A$  such that  $(a, b) \in R$ , if book 'a' costs more and contains fewer pages than book 'b'. Is  $R$  an Equivalence Relation or a Partial Order Relation? Justify.

(2)

### Section-B

5. (a) Prove that total number of permutations of  $n$  different things taken not more than  $r$  at a time, when each thing may be repeated any number of times is  $n(n^r - 1)/(n - 1)$ . (5)
- (b) Consider  $a \in \mathbb{R}$  as a constant real number. Assume  $G = \{a^n : n \in \mathbb{Z}\}$ . Prove that  $G$  is an abelian group under usual multiplication. (5)
6. (a) Solve the recurrence relation  $a_r - 2a_{r-1} + a_{r-2} = 2^r$ ,  $r \geq 2$ , by the method of generating function satisfying the boundary conditions  $a_0 = 2, a_1 = 1$ . (6)
- (b) Define order and size of a graph. Describe Complement and Subgraph of a graph giving examples. (4)
7. (a) Discuss the Breadth-First Traversal technique using the given graph. (6)



- (b) What do you understand by counting techniques? Explain. (2)
- (c) Define an algebraic structure. Differentiate between a ring and a field. (2)