Exam.Code:0933
Sub. Code: 33745

2124

B.E. (Electrical and Electronics Engineering)
Third Semester
BS-EE-305: MATH-III

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.*

x-x-x

1. (a) Explain the differences between a consistent system and inconsistent system of a linear system of equations with suitable examples.

(b) Define basis and dimension of a vector space. Explain the relation between them.

(c) State Cayley-Hamilton theorem. Explain its significance and applications in linear algebra.

(d) Explain the concept of differentiability in complex function and how it differs from real differentiability.

(e) State Laurent's theorem. Find Laurent's series of $f(z) = \frac{1}{z-2}$ about $z = 2$.

SECTION-A

2. (a) Discuss the advantages of representing a system of linear equations in triangular form for computational purposes. Analyze the consistency of the following system using row echelon form:

$$4x - y + z = 3;\ 2x + y + 3z = 7;\ 6x - 3y + 2z = 5.$$

(b) Determine the values of k for which the system: $x - ky + z = 0;\ kx + 3y - kz = 0;$

$3x + y - z = 0$ has (i) only trivial solution, (ii) non-trivial solution.

(c) Define a vector space. Examine whether the set of all polynomials of degree less than or equal to three is a vector space. Justify. $(4 + 3 + 3)$

3. (a) Find condition on a, b, c so that the vector $v = (a, b, c) \in R^3$ belongs to the space generated by $v_1 = (2,1,0);\ v_2 = (1,-1,2);\ v_3 = (0,3,-4)$.

(b) Examine whether the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ is diagonalizable or not? If yes, then find the invertible matrix P such that $P^{-1}AP$ is diagonal matrix.

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4. (a) State rank-nullity theorem. Verify the the same for the the linear transformation $T: R^3 \to R^3$ be defined as $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$.

(b) Let T be a linear operator on R^3 defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$. Find the matrix of T relative to the basis $B = \{(1,1,1), (1,1,0), (1,0,0)\}$. Verify that

$[T; B]\,[v\,; B] = [T(v); B]\ \forall\, v \in R^3$.

SECTION-B

5. (a) Derive the relationship between trigonometric functions and hyperbolic function in the complex domain. Find all values of z such that $\sinh z = e^{\frac{\pi i}{3}}$. Also, prove that

$|\sinh z|^2 = \sinh^2 x + \sin^2 y$. (4 + 3 + 3)

(b) Define the complex exponential function along with its various properties. Discuss its continuity, differentiability and analyticity.

(c) Prove that the real and imaginary parts of an analytic function are harmonic functions.

6. (a) If $u(x, y)$ is a harmonic function, then prove that $w = u^2$ is not harmonic, unless u is a constant function. (4 + 3 + 3)

(b) Find the conjugate harmonic function of $u = \left\{r + \frac{1}{r}\right\} \cos\theta,\ r \neq 0$

(c) Explain the different types of isolated singularities with suitable examples.

7. (a) State residue theorem. Hence, evalaute $\oint \frac{dz}{(z^2+4)^2}$ along a curve $|z - i| = 2$.

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the images of $z = 4$ and $|z| < 1$.

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