Exam.Code:0939
Sub. Code: 33846

2124
B.E. (Mechanical Engineering)
Third Semester
ASM-301: Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part

x-x-x

1. Answer the following: (5 × 2 = 10)

(a) Check the linear dependence of following vectors: $u = (1, 1, 0)$, $v = (1, 3, 2)$ and $w = (4, 9, 5)$.

(b) Let $F : R^4 \to R^3$ be a linear map defined by

$$F(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$$

Find the dimension of the image of F.

(c) Find th eigen values of the matrix $B = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$

(d) Show that $\log \frac{x + iy}{x - iy} = 2i \tan^{-1}(y/x)$.

(e) Determine the pole and residue at the pole of the function $f(z) = z/(z-1)$.

PART A

2. (a) For the following matrix A, find an orthogonal matrix P and a diagonal matrix D such that $D = P^{-1}AP$ (5)

$$A = \begin{bmatrix} 5 & 4 \\ 4 & -1 \end{bmatrix}$$

(b) For the following linear operator $T : R^2 \to R^2$, find all the eigenvalues and a basis for the eigenspace. (5)

$$T(x, y) = (3x + 3y, x + 5y)$$

3. (a) Let $F : R^4 \to R^3$ be the linear mapping defined by (6)

$$F(x, y, z, t) = (x - y + z + t, x + 2z - t. x + y + 3z - 3t)$$

Find a basis and the dimension of the Image and the Kernal of F.

(b) Let $H : R^2 \to R^3$ be defined by $H(x, y) = (x + y, x - 2y, 3x + y)$. Check whether H is a singular or a nonsingular linear map. (4)

4. (a) Find a basis and dimension of the solution space W of the following homogeneous system: (5)

$$x + 2y - 2z + 2s - t = 0$$
$$x + 2y - z + 3s - 2t = 0$$
$$2x + 4y - 7z + s + t = 0$$

P.T.O.

(b) Solve the following system of equations: (5)

$$x + 2y - z = 3$$

$$x + 3y + z = 5$$

$$3x + 8y + 4z = 17$$

PART B

5. (a) Solve the equation $\sinh(z) = i$. (4)

(b) Verify that $z = i$ is the only solution of the equation $\log(z) = (\pi/2)i$. (3)

(c) Find the value of $(1 - i)^{1+i}$. (3)

6. (a) Examine the continuity of the function $f(z)$ defined by (4)

$$f(z) = \begin{cases} \dfrac{Im(z)}{|z|} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$$

at $z = 0$.

(b) Show that the function defined below is not analytic at $z = 0$, althought, Cauchy-Riemann equations are satisfied at the point. How would you explain this. (6)

$$f(z) = \begin{cases} e^{-z^{-4}} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$$

OR

7. (a) Using Residue theorem, evaluate the following integral (6)

$$\frac{1}{2\pi i} \int_C \frac{e^{zt}}{z^2(z^2 + 2z + 2)} dz$$

(b) Determines the poles of the function $f(z) = 1/(z^4 + 1)$. (4)

x-x-x