Exam.Code:1014
Sub. Code: 35201

2124

M.E. (Mechanical Engineering)
First Semester
MME-101: Advanced Engineering Mathematics

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, selecting atleast two questions from each Section. Use of Simple calculator is allowed. All questions carry equal marks.

x-x-x

SECTION-A

1. (a) Locate and classify the regular and singular point(s) of the differential equation: $x(x+1)^2(x-1)\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + (x^3-1)y = 0$. Find its series solution about $x = 0$.

(b) Find two linearly independent solutions of $(x+2)^2\frac{d^2y}{dx^2} + (x+2)\frac{dy}{dx} - 4y = 0$ about $x = -2$. Also find its general solution.

2. (a) State and prove Rodrigue's formula. How is this formula useful?

(b) Prove that $(1-x^2)P_{n-1}'(x) = n[x P_{n-1}(x) - P_n(x)]$.

3. (a) Prove that: $\frac{d}{dx}[J_n^2(x) + J_{n+1}^2(x)] = 2\left[\frac{n}{x} J_n^2(x) - \frac{(n+1)}{x} J_{n+1}^2(x)\right]$.

(b) State and prove the orthogonality of Bessel's functions.

4. (a) Define modified Bessel functions of first and second kinds. Solve:

$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \frac{1}{2}y = 0$ in terms of Bessel's function.

(b) Find the eigenvalues and eigen functions of the Strum-Liouville problem:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + (\lambda+1)y = 0, y(1) = 0, y(0) = 0.$$

SECTION-B

5(a) Solve the IVP: $\frac{dy}{dx} = z + x;\ \frac{dz}{dx} = y^2 + x + z,\ y(0) = 0, z(0) = 0$ with the help of Picard method. Obtain only first three approximation and find the values of $y(0.1)$ and $z(0.1)$.

(b) Solve the second order IVP: $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y^2 = \sin x$ with initial conditions: $y(0) = 1, \frac{dy}{dx}(0) = 1$ using Runge-Kutta method of order 4 to find $y(0.5)$.

P.T.O.

6. Solve the BVP: $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x + 4; 0 \leq x \leq 0.6$ subject to boundary conditions: $\frac{dy}{dx}(0) = 1, y(0.6) = 1.96$, by taking step size $h = 0.2$. Use central difference approximation for the differential equation and derivative boundary condition.

7. (a) Describe the similarities and difference between explicit and implicit finite difference methods.

   (b) Solve the Laplace equation on a rectangular domain using finite difference method with Dirichlet boundary conditions.

8. Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subjected to the conditions: $u(0, t) = 0$ and $u(x, 0) = \sin \pi x$.

x-x-x