(i) Printed Pages: 4 Roll No. ....

(ii) Questions : 7 Sub. Code : | 3 | 3 | 2 | 4 | 1 |

Exam. Code : | 0 | 9 | 0 | 5 |

B.Engg. 1st Year (1st Semester)

(2124)

CALCULUS (Common to All Streams)

Paper : ASM-101

Time Allowed : Three Hours] **[Maximum Marks : 50**

Note :— Attempt **FIVE** questions in all selecting **TWO** questions from each part. First question is compulsory. Use of non programmable calculator is allowed.

1. (a) Check the convergence of the series : $\sum_{n=3}^{\infty} \frac{\ln n}{\ln(\ln n)}$.

(b) The first term of the sequence is $x_1 = \cos(1)$. The next terms are $x_2 = x_1$ or $\cos(2)$, whichever is larger; and $x_3 = x_2$ or $\cos(3)$, whichever is larger. In general,

$$x_{n+1} = \max\{x_n, \cos(n+1)\}.$$

Check the convergence or divergence of the sequence.

(c) Find the domain, range of the function $f(x, y, z) = y^2 + z^2$. Is the domain open/closed, bounded/unbounded ? Also draw its level surfaces.

(d) Find tangent plane and normal line of the level surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $P(1, 2, 4)$.

(e) Find the volume of the smaller region cut from the solid sphere $p \leq 2$ by the plane $z = 1$. 5×2=10

PART—A

2. (a) Find all values of x for which the following series is convergent :

$$\sum_{n=1}^{\infty} \frac{n\,x^n}{(n+1)\,(2x+1)^n}.$$ 5

(b) (i) Check the convergence of the series : $\sum_{n=2}^{\infty} \frac{\ln_n(n!)}{n^3}$. 3

(ii) Evaluate the limit, if it exists : $\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$. 2

3. (a) Let $T = f(x, y)$ be the temperature at the point (x, y) on the circle $x = \cos(t)$, $y = \sin(t)$, $0 \leq t \leq \pi$ and suppose that

$$\frac{\partial T}{\partial x} = 8x - 4y, \quad \frac{\partial T}{\partial y} = 8y - 4x.$$

(i) Find where the maximum and minimum temperatures on the circle occur by examining the derivatives dT/dt and d^2T/dt^2.

(ii) Suppose that $T = 4x^2 - 4xy + 4y^2$. Find the maximum and minimum values of T on the circle. 4

(b) Find the area of the region enclosed by the lines/curves :

$x = y^2 - 1$ and $x = |y|\sqrt{1-y^2}$. 3

(c) Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis, and on the right by the line $x = 2$ about y-axis. 3

4. (a) Suppose that $f(x, y, w, z) = 0$ and $g(x, y, w, z) = 0$ determine z and w as differentiable functions of the independent variables x and y, and suppose that $f_z g_w - f_w g_z \neq 0$.

Show that :

$$\left(\frac{\partial z}{\partial x}\right)_y = \frac{f_x g_w - f_w g_x}{f_z g_w - f_w g_z} \text{ and } \left(\frac{\partial w}{\partial y}\right)_x = \frac{f_z g_y - f_y g_z}{f_z g_w - f_w g_z}.$$ 5

(b) Using Lagrange's multiplier method, find the point closed to the origin on the curves of intersection of the plane $2y + 4z = 5$ and the cone $z^2 = 4x^2 + 4y^2$. 5

PART—B

5. (a) Using triple integral, find the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cylinder $x^2 + y^2 = 1$. 5

(b) Let D be the region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above the paraboloid $z = 2 - x^2 - y^2$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the following order of integration and hence evaluate the volume of the solid D.

(i) $dz\ dr\ d\theta$

(ii) $d\theta\ dz\ dr$. 5

6. (a) Show that the differential form in the integral is exact. Then evaluate the integral :

$$\int_{(1,2,1)}^{(2,2,1)} \left[(2x \ln y - yz)dx + \left(\frac{x^2}{y} - xz \right) dy - xy\, dz \right]$$ 5

(b) Use divergence theorem to calculate the outward flux of F across the boundary of the region D : The region inside the solid cylinder $x^2 + y^2 \leq 4$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$ and

$$F = y\hat{i} + xy\hat{j} - z\hat{k}.$$ 5

7. (a) Find T, N, B and the equations for the osculating, normal and rectifying planes at $t = 0$.

$$r(t) = (\cos(t))\hat{i} + (\sin(t))\hat{j} + t\hat{k},\ t = 0.$$ 7

(b) A particle traveling in a straight line is located at the point (1, –1, 2) and has speed 2 at time $t = 0$. The particle moves toward the point (3, 0, 3) with constant acceleration $2\hat{i} + \hat{j} + \hat{k}$.

Find its position vector $r(t)$ at time t. 3