Exam.Code:0918
Sub. Code: 6408

2053

B.E. (Computer Science and Engineering)
Sixth Semester
CS-602: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: ***Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section.***

x-x-x

1 (a) Prove that every subset of a linearly independent set of vectors of the vector space V(F) is linearly independent.

(b) Show that the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$T(x, y, z) = (x + z, \ -x + 2y + z, \ y + z)$ is neither one-one nor onto.

(c) Prove that 0 is an eigen value of a square matrix A iff A is singular matrix.

(d) An integer is chosen at random from 1 to 200 (inclusive). What is the probability that the integer is divisible by 6 or 8?

(e) Find the expectation of the number on a die when thrown.

$(5 \times 2 = 10)$

Section–A

2 (a) Show that the only real value of λ for which the equations

$$x + 2y + z = \lambda x; \ \ x + y + 2z = \lambda y; \ 2x + y + z = \lambda z,$$

have a non-trivial solution is 4. Also solve the equations for $\lambda = 4$.

(b) Prove that any linearly independent set of a finite dimensional vector space can be extended to form the basis of the vector space.

(5+5)

3 (a) Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$.

Is A diagonalizable? If yes, obtain the invertible matrix P such that $P^{-1}AP$ is a diagonal matrix.

(b) Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation defined as

$T(x, y, z) = (x - 2y + z, \ x - 3y, \ y + 2z, \ x + 4y - 3z)$ with respect to the usual ordered basis for $\mathbb{R}^3$ and $\mathbb{R}^4$.

$(5 + 5)$

4(a) Find the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(1, 2) = (3, \ -1, 5)$ and $T(1, 0) = (2, 1, \ -1)$. Also verify Rank – Nullity theorem for T.

(b) Let $T: V \to W$ be a linear transformation where $\dim V = \dim W$. Prove that T is non – singular if and only if T is onto.

$(5 + 5)$

P.T.O.

Section – B

5(a) The chances that doctor A will diagnose a disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly? (5 + 5)

(b) A and B throw alternatively with a pair of balanced dice. A wins if he throws a sum of six points before B throws a sum of seven points, while B wins if he throws a sum of seven points before A throws a sum of six points. If A begins the game, show that his probability of winning is 30/61.

6(a) Let the random variable X assume the value r with the probability law:

$$P(X = r) = q^{r-1}p; r = 1, 2, 3, . . .$$

(5 + 5)

Find the moment generating function of X and hence its mean and variance.

(b) An irregular six faced die is thrown and the expectation that in 10 throws it will give five even numbers is twice the expectation that it will give four even numbers.How many times in 10,000 sets of 10 throws each, would you expect it to give no even number?

7(a) In two sets of variables X and Y with 50 observations each, the following data were observed: $\overline{X} = 10,\ \sigma_X = 3,\ \overline{Y} = 6,\ \sigma_Y = 2$ and $r(X, Y) = 0.3$. But on subsequent verification it was found that one value of $X\ (= 10)$ and the corresponding value of $Y\ (= 6)$ were inaccurate and hence weeded out. With the remaining 49 pair of values, how is the original value of r affected?

(b) Obtain the equation of two lines of regression for the following data

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

Also obtain the estimate of X for Y = 70. (5+5)

x-x-x