Exam. Code: 0927
Sub. Code: 6571

2063

B.E. (Electronics and Communication Engineering)
Third Semester
MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.

x-x-x

1 (a) Examine whether $(1, -3, 5)$ belongs to the linear space generated by $S = \{(1, 2, 1), (1, 1, -1), (4, 5, -2)\}$ or not?

(b) Prove that a linear transformation $T: V \to W$ is non-singular if and only if T is one–one.

(c) Verify Cayley–Hamilton theorem for square matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$.

(d) Can a function be differentiable at a point without being analytic there? If yes, give an example.

(e) What is a conformal mapping? Find the points at which $w = \sin z$ is not conformal.

$(5 \times 2 = 10)$

Section – A

2 (a) Let $V_n(\mathbb{R})$ denote the set of all polynomials in x of degree $\leq n$ (a non-negative integer) and the zero polynomial. Prove that $V_n(\mathbb{R})$ is vector space over the field $\mathbb{R}$ under the usual addition and scalar multiplication of polynomials.

(b) Let V be the vector space of all 2×2 symmetric matrices over $\mathbb{R}$. Find a basis and the dimension of V.

3 (a) Define null space, image space and their spans. Find a linear map $T: \mathbb{R}^4 \to \mathbb{R}^3$ whose null space is generated by $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$.

(b) If $B_1 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $B_2 = \{(1, 0, 1), (-1, 2, 1), (2, 1, 1)\}$ are basis of $\mathbb{R}^3$, then find the transition matrices P and Q from basis B_1 to B_2 and B_2 to B_1 respectively.

4(a) Let T be a linear operator on $\mathbb{R}^3$ defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}.

(b) Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$. Is A diagonalizable? If yes, obtain the invertible matrix P such that $P^{-1}AP$ is a diagonal matrix.

Section – B

5 (a) Find all solutions of $\cos z = i$.

(b) Find $u(x, y)$ and $v(x, y)$ such that $\tan z = u(x, y) + iv(x, y)$. Determine where these functions are defined, and show that they satisfy the Cauchy-Riemann equations for these points (x, y).

(c) Write down the differences and similarities between sin z and sin x. Is sinz bounded? If yes, prove it.

6 (a) Find the Laurent series expansion of $\frac{1}{z(e^z-1)}$ for the region $0 < |z| < 2\pi$.

(b) Evaluate the integral $I = \int_0^{2\pi} \frac{d\theta}{\pi + 3\cos\theta}$.

7(a) Define fixed and critical points of a mapping with suitable examples. Let f be a linear fractional transformation with three fixed points. Prove that f is the identity mapping.

(b) Determine all singularities of the function $f(z) = \frac{z-i}{z^2+1}$ and classify each singularity as removable, a pole of a certain order, or an essential singularity.

(c) Find all the values of $(1 - i)^{1+i}$.

(3+3+4)

x-x-x