Exam.Code:0933
Sub. Code: 6660

2063

B.E. (Electrical and Electronics Engineering)
Third Semester
BS-EE-305: MATH-III

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.

x-x-x

1 (a) Define vector subspace. Is the set of all points on the line $x + y = 1$ in the xy − plane, a vector subspace of R^2 with respect to usual vector addition and scalar multiplication.

(b) Define singular and non-singular transformation. Prove that there is no non-singular linear transformation from R^4 to R^3.

(c) Define algebraic and geometric multiplicity of an eigenvalue of a square matrix. What is a defective eigenvalue? Find the same for $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$. $(5\times2=10)$

(d) List out differences and similarities between cos x and cos z.

(e) Define residue. Explain its role in complex integration.

SECTION-A

2. (a) Find the column rank of the matrix: $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$. $(3+4+3)$

(b) Let R be the field of real numbers. Determine which of the following are sub-spaces of $R^3(R)$?

(i) all the vectors of the form $\{(x, 2y, 3z)\}$, (ii) all the vectors of the form (x,x,x), (iii) $\{(x, y, z): x, y, z \text{ are rational numbers}\}$.

(c) Solve the system: $2x + z = 3;\ x - y + z = 1;\ 4x - 2y + 3z = 3$ by Gauss elimination method.

3. (a) Prove that a transformation $T: R^3 \to R^3$ defined by $T(x,y,z) = (x - y, y - z, z - x)$ is a linear transformation. Also find range space and null space of T.

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(b) Verify rank-nullity theorem for the linear transformation $T:R^4 \to R^3$, defined by $T(x,y,z,t)=(x+y, y-z, z-t)$

4. (a) Examine whether $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & 2 & 0\end{bmatrix}$ is diagonalizable or not? If yes, obtain the matrix P such that $P^{-1}AP$ is a diagonalizable.

(b) Let $T:R^3 \to R^2$ be a linear transformation defined by $T(x,y,z)=(2x+y-z, 3x-2y+4z)$. Find the matrix of T with respect to ordered basis $B_1 = \{(1,1,1),(1,1,0),(1,0,0)\}$ and $B_2 = \{(1,3),(1,4)\}$ of R^3 and R^2 respectively.

SECTION-B

5. (a) Distinguish between the concept of differentiability and analyticity of a function at a point. Give an example of a function which is (i) differentiable but not analytic at a point, (ii) differentiable as well as analytic? Also state necessary and sufficient conditions for analyticity.

(b) If f(z)is analytic, then prove that $|f(z)|$ is not a harmonic function.

(c) Solve sinhz = 2i . (4+3+3)

6 (a) State Laurent's theorem. Find all possible Taylor's and Laurent series expansions for the function $f(z)=\dfrac{1}{z(1-z^2)}$.

(b) Define residue. Evaluate $\int_C \dfrac{1}{\sinh z}dz$, where C is the circle $|z|=4$ using residue theorem.

7. (a) Discuss the mapping $w = z^2$. Under this mapping, find the image of a region bounded by the lines $x = 1, y = 1$ and $x + y = 1$.

(b) Evaluate the integral using contour integration: $I=\int_0^{2\pi}\dfrac{\cos\theta}{5+4\cos 2\theta}d\theta$.

x-x-x