Exam.Code:0939
Sub. Code: 6706

2063
B.E. (Mechanical Engineering)
Third Semester
ASM-301: Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.

x-x-x

1 (a) Under what condition on the scalar $b \in \mathbb{R}$, are the vectors $(b, 1, 0)$, $(1, b, 1)$ and $(0, 1, b)$ in $\mathbb{R}^3(\mathbb{R})$ linearly independent?

(b) Show that there is no non–singular linear transformation from $\mathbb{R}^4$ to $\mathbb{R}^3$.

(c) If A is non–singular matrix, then prove that the eigen values of A^{-1} are the reciprocal of the eigen values of A. $(5 \times 2 = 10)$

(d) Is the function $f(z) = 3\pi^2/(z^3 + 4\pi^2 z)$ analytic? Justify.

(e) What is residue? What is the role of residue in integration?

Section – A

2 (a) Prove that linear span of any subset S of a vector space $V(F)$ is a subspace of $V(F)$.

(b) Let M and N be two subspaces of $\mathbb{R}^4(\mathbb{R})$, where $M = \{(a, b, c, d): b + c + d = 0\}$ and $N = \{(a, b, c, d): a + b = 0, c = 2d\}$. Find a basis and dimension of M, N and $M \cap N$.

3 (a) Show that the mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined as $T(x, y, z) = (x - y, y - z, z - x)$ is a linear transformation. Also find range space of T and null space of T.

(b) If $V(F)$ and $W(F)$ are two finite dimensional vector spaces over the same field F such that $\dim V = \dim W$, then prove that V and W are isomorphic to each other.

4(a) Prove that the eigen vectors corresponding to distinct eigen values of a matrix are linearly independent.

(b) State Cayley–Hamilton theorem and verify the same for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$.

Hence find A^{-1}.

P.T.O.

Section – B

5(a) Let $f(z) = \begin{cases} \frac{z^5}{|z|^4} & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$. Show that Cauchy-Riemann equations are satisfied at $z = 0$, but f is not differentiable at 0.

(b) Find all solutions of $e^z = 1 + 2i$.

(c) Prove that $w = \cos z$ is not a bounded function.

6 (a) Explain the difference between Taylor's series and Laurent's series. Find the Laurent series expansion of $f(z) = \frac{1}{z(1-z^2)}$ and determine the precise region of its convergence.

(b) State Cauchy residue theorem. Use it to evaluate the integral $I = \int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = \frac{3}{2}$.

7(a) Explain different types of singularities with suitable examples. Determine all singularities of the function $f(z) = \frac{\sin z}{\sinh z}$ and classify each singularity as removable, a pole of a certain order, or an essential singularity.

(b) Evaluate the integral $I = \int_0^{2\pi} \frac{d\theta}{\pi + 3\cos\theta}$.

x-x-x