Exam.Code:0918
Sub. Code: 6408

2023

B.E. (Computer Science and Engineering)
Sixth Semester
CS-602: Linear Algebra and Probability Theory

Time allowed: 3 Hours
Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.

x-x-x

Q.No:1 (a) Let W_1 and W_2 be any two subspaces of a vector space $V(F)$, prove that $W_1 + W_2 = \{x + y : x \in W_1, y \in W_2\}$ is subspace of $V(F)$.

(b) If v_1, v_2, v_3 are linearly independent vectors of $V(F)$, is the set of vectors $v_1 + v_2, v_1 - v_2, v_1 - 2v_2 + v_3$ linearly independent or not? Justify.

(c) Let $T : \mathbb{R}^3(\mathbb{R}) \to \mathbb{R}^3(\mathbb{R})$ defined as $T(x, y, z) = (3x, x - y, 2x + y + z)$. Prove that T is invertible and find T^{-1}.

(d) For any two events A and B such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P($not A or not B $= \frac{1}{4}$. Check if the events A and B are independent.

(e) The joint probability density function of continuous random variables X and Y is given by

$$f(x, y) = \begin{cases} cxy, & 0 < x < 4,\ 1 < y < 5 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the value of c. (2 × 5)

Section: A

Q.No:2 (a) Find a basis and dimension of the solution space S of the following linear equations

$$\begin{aligned} x + 2y - 2z + t &= 0, \\ 2x + 4y - 2z + 4t &= 0, \\ 2x + 4y - 6z &= 0, \\ 3x + 6y - 8z + t &= 0. \end{aligned}$$

(b) Let V be a vector space over the field F. Prove that the set S of non-zero vector $v_1, v_2, ..., v_n \in V$ is linearly dependent iff some vector, say v_k, $2 \leq k \leq n$, can be expressed as the linear combination of the preceding vectors of the set S.

Q.No:3 (a) Let V be the vector space of all polynomials over $\mathbb{R}$ of degree less than or equal to 3. Let W be a subspace of V generated by the polynomials $t^3 - 2t^2 + 4t + 1$, $2t^3 - 3t^2 + 9t + 1$, $t^3 + t + 1$ and $2t^3 - 5t^2 + 7t + 1$. Find a basis and dimension of the subspace W. Also extend it to a basis of V.

P.T.O.

(b) State Cayley-Hamilton theorem for a matrix and verify it for the matrix

$$\begin{bmatrix} 3 & -2 \\ 4 & -4 \end{bmatrix}.$$

Q.No:4 (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (3x+y+4z, 2y+6z, 5z)$. Find the matrix of T with respect to standard basis of $\mathbb{R}^3(\mathbb{R})$. Is the matrix diagonalizable? Justify.

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be linear transformation defined by $T(x, y) = (x + y, x - y, y)$. Find (i) Range Space and Rank of T and (ii) Null Space and Nullity of T.

Section: B

Q.No:5 (a) Prove that for n events $A_1,\ A_2, ..., A_n$,

$$P(\cap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n - 1).$$

(b) A deck of playing cards is found to contain 51 cards. If the first thirteen examined cards are all red, what is the probability that the missing card is black?

Q.No:6 (a) If a random variable X has probability density function

$$f(x) = \frac{c}{x^2 + 1}, \quad -\infty < x < \infty.$$

(i) Find the value of the constant c, (ii) $P(\frac{1}{3} \leq x^2 \leq 1)$.

(b) If the random variable X follows binomial distribution with parameters n and p, prove that $P(X = \text{even}) = \frac{1}{2}[1 + (q - p)^n]$.

Q.No:7 (a) Find the mean and variance of a normal distribution.

(b) The random variables X and Y have the joint probability density function

$$f(x, y =) \begin{cases} x + y, \ 0 < x < 1, \ 0 < y < 1 \\ \quad 0, \text{elsewhere.} \end{cases}$$

Find the coefficient of correlation between X and Y.

x-x-x