

2023

B.E. (Mechanical Engineering)

Fourth Semester

MEC-406: Numerical Analysis

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section. Use of simple calculator is allowed. All questions carry equal marks.

x-x-x

1. (a) Explain the difference between accuracy and precision in numerical analysis with suitable examples. Explain an error and its different types.
- (b) State intermediate value property of a continuous function? Explain the difference between bracketing methods and open methods with suitable examples.
- (c) Explain the pitfalls of elimination method in solving linear algebraic system. Explain the techniques for improving the solution.
- (d) Define numerical differentiation and integration. What is the difference between them?
- (e) Explain the difference between IVP and BVP. Why do we need numerical methods for solving them? Explain Euler's methods for solving IVP.

SECTION-A

2. (a) Explain error in series approximation. Compute $\log_e(1.02)$ truncating after the third term. Find the error.
- (b) State the sufficient conditions for the convergence of fixed point iteration method. Find the value of $\sqrt{90}$ using fixed point iteration method correct to four decimal places.
3. (a) Prove that the LU decomposition method fails to solve the system of equations: $x + y - z = 2$; $2x + 2y + 5z = -3$; $3x + 2y - 3z = 6$. Justify, why it fails?

- (b) Explain condition number. Find the same for the matrix: $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$.

4. (a) Find the power fit $y = a x^m$ for the data:

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(2)

- (b) Explain interpolation and inverse interpolation along with applications. Find the value of x for $f(x) = 10$ using the following table:

x	2	3	4	5
$f(x)$	8	27	64	125

SECTION-B

5. (a) Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using trapezoidal rule and Romberg's method.

- (b) Let $f(x) = e^x$. Using a central difference formula of $O(h^2)$, find $f^{(11)}(1)$.

Improve this value using Richardson's extrapolation by taking

$$h = 0.1 \text{ and } h = 0.05.$$

6. (a) Solve: $\frac{dy}{dx} = x + z$, $y(0) = 0$; $\frac{dz}{dx} = x - y$, $z(0) = 1$ for $x = 0.1$ by Runge-Kutta method.

- (b) Solve the BVP: $y^{(11)} = x + y$, $y(0) = 1$, $y(1) = 1$ by finite difference method.

7. Solve: $2 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(0, t) = 0$, $u(4, t) = 0$, and $u(x, 0) = x(4 - x)$, choosing $h = k = 1$ and using Bender-Schmidt formula, find the values up to $t = 5$.