Exam.Code:0906
Sub. Code: 6243

2023

B.E., Second Semester

ASM-201: Differential Equations and Transforms

(Common to Bio-Tech, EEE, IT, CSE, Civil, MEC, ECE)

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.*

x-x-x

1. (a) Explain geometrically the meaning of an ordinary differential equation? Define the terms auxiliary equation, complementary function and particular integral with suitable example.

(b) Define Laplace transformation along with its applications. What are the minimum requirements for a valid transform along with suitable example.

(c) State Dirichlet's conditions for the Fourier series. Are they necessary? Write Dirichlet's conditions for the existence of Fourier series of a function f(x) in $(\alpha, \alpha + 2\pi)$.

(d) Define Fourier integral. Write down any three properties of Fourier integrals.

(e) What is the difference between the order and degree of a partial differential differential equation? List some standard partial differential equations with examples.

SECTION-A

2. (a) Discuss the geometrical interpretation of $\frac{dy}{dx} = -\frac{9}{4}\frac{x}{y}$.

(b) Solve: $\left(x\,y^2 - e^{x^{\frac{1}{3}}}\right) dx - x^2 y\, dy = 0.$

(c) Solve by method of variation of parameters: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x.$

3. (a) Find the power series solution of the equation:

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0 \text{ in powers of x about x} = 0.$$

(b) Define unit step function. Find the Laplace transform of:

$$f(t) = u(t-3)\,(2t^2 + 3t - 1)$$

4. (a) State convolution theorem. Find $L^{-1}\left(\frac{1}{\sqrt{s}\ (s-1)}\right)$ using convolution theorem.

(b) Solve: $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = t,\ y(0) = 0$ and $\frac{dy}{dt}(0) = 0$.

SECTION-B

5. (a) Expand $f(x) = |\sin x|$ in Fourier series, $-\pi \leq x \leq \pi$.

(b) Find the complex Fourier series for the function f(x) defined by

$$f(x) = e^x,\ -\pi < x < \pi.$$

6. (a) Express $f(x) = e^{-x} + e^{-2x},\ x > 0$ as a Fourier sine and cosine integral.

(b) Solve the partial differential equation: $(x^2 + y^2 + z^2)\,p - 2\,xy\,q = -2xz$.

7. (a) Form a partial differential equation by elimination of the arbitrary functions from $f(xy + z^2,\ x + y + z) = 0$.

(b) Solve $\frac{\partial u}{\partial t} = k\,\frac{\partial^2 u}{\partial t^2}$, $0 < x < 2\pi$ with the condition:

$$u(x, 0) = x^2,\ u(0, t) = u(2\pi, t) = 0.$$

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