Exam.Code:1014
Sub. Code: 7431

2023

M.E. (Mechanical Engineering)

First Semester

MME-101: Advanced Engineering Mathematics

Time allowed: 3 Hours Max. Marks: 50

***NOTE**: Attempt five questions in all, selecting atleast two questions from each Unit. Use of simple calculator is allowed.*

x-x-x

UNIT - I

I. Using Frobenius method, find series solution of the differential equation:

$(x^2 - x)y'' - xy' + y = 0$ about its singular point. (10)

II. (a) Solve the Legendre's differential equation:

$(1 - x^2)y'' - 2xy' + n\,(n+1)y = 0$. (5)

(b) State and prove Rodrigues's formula. (5)

III. Derive a general expression of Bessel's function $J_v(x)$ for $v \geq 0$ and hence evaluate $J_{1/2}(x)$. (10)

IV. (a) State Strum-Liouville's problem. Also, find eigen values and eigen vectors of $y'' + \gamma y = 0$, $y(0) = 0,\ y(\pi) = 0$. (5)

(b) Derive orthogonality relation for Bessel's functions. (5)

UNIT - II

V. Solve the boundary value problem using Finite-Difference method for $x = 0.5$, for $\frac{d^2y}{dx^2} + y + 1 = 0,\ y(0) = y(1) = 0$. (10)

VI. Apply Runge-Kutta's method of fourth order to find an approximate value of y for $x = 0.2$ in steps of 0.1 for $\frac{dy}{dx} = \sqrt{(x+y)},\ y(0) = 1$. (10)

VII. Find the values of $u(x,t)$ satisfying the parabolic equation $u_t = 4\,u_{xx}$ and the boundary conditions $u(0,t) = u(8,t) = 0$ and $u(x,0) = 4x - \frac{1}{2}x^2$ at the points $x = i : i = 0,1,2,\ldots,8$ and $t = \frac{1}{8}j : j = 0,1,2,\ldots,5$. (10)

VIII. Evaluate the pivotal values of $u_{tt} = 16\,u_{xx}$ upto t=1.25. The boundary conditions are $u(0,t) = u(5,t) = 0$ and initial conditions are $u_t(x,0) = 0, u(x,0) = x^2(5-x)$. (10)

x-x-x